

# An Overview of Implication Based Interval Valued on Fuzzy K-Ideals and Its Applications

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## ABSTRACT

Ideals of semirings play a central role in the structure theory and are useful for many purposes. Henriksen defined in a more restricted class of ideals in semirings, which is called the class of k-ideals, with the property that if the semiring R is a ring then a complex in R is a k-ideal if and only if it is a ring ideal.

The Aim of this project is to introduce and study new different sorts of interval-valued fuzzy k-ideals of semiring and to investigate the new aspects of related properties. In particular interval-valued fuzzy k-ideals with thresholds are investigated

**KEYWORDS:** Semiring, k-Ideals,  $\tilde{t}$ -tautology, Ordinary interval, Consequence

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## INTRODUCTION:

Set theoretic multi-valued logic is a special case of fuzzy logic such that the truth values are linguistic variables (or terms of the linguistic variables truth).by  $\wedge, \vee, -, \rightarrow$  can be applied in fuzzy proposition P. In the following, we show a correspondence between fuzzy logic and set-theoretical notions.

$$[x \in F] = \mu_{F(x)}^{\sim},$$

$$[P \wedge Q] = \min\{[P], [Q]\},$$

$$[P \rightarrow Q] = \min\{[1, 1], [1, 1] - [P] + [Q]\}, \forall x$$

$$P(x) = \inf[P(x)],$$

$$[x \notin F] = [1, 1] - \mu_F^{\sim}(x),$$

$$[P \vee Q] = \max\{[P], [Q]\},$$

$\models P$  if and only if  $[P] = [1, 1]$  for all valuations.

We show some of important implication operators, where  $\alpha$  and  $\beta$  is the degree of membership of the consequence and I the resulting degree of the truth for the implications.

Early zad	$I_m(\alpha, \beta) = \max\{1 - \alpha, \min\{\alpha, \beta\}\},$
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Lukasiewicz	$I_a(\alpha, \beta) = \min\{1, 1 - \alpha + \beta\},$
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Standard star (Godel)	$I_g(\alpha, \beta) = \begin{cases} 1 & \text{if } \alpha \leq \beta \\ \beta & \text{Otherwise} \end{cases}$
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Contraposition of Godel	$I_{cg}(\alpha, \beta) = \begin{cases} 1 & \text{if } \alpha \leq \beta \\ 1 - \alpha & \text{Otherwise} \end{cases}$
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Gaines Rescher	$I_{gr}(\alpha, \beta) = \begin{cases} 1 & \text{if } \alpha \leq \beta \\ 0 & \text{Otherwise} \end{cases}$
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Kleene-Dienes

$$I_b(\alpha, \beta) = \max \{1, 1 - \alpha, \beta\},$$

# DEFINITIONS:

An interval-valued fuzzy set  $F$  of  $R$  is called an **interval-valued fuzzifying k-ideal** of  $R$ .

If it satisfies the following

- 1) For any  $x, y \in R$ ,

$$\models \{[x \in F] \wedge [y \in F]\} \rightarrow [x+y \in F],$$

- 2) For any  $x, a \in R$ ,  
 $\models \{[x \in F]\} \rightarrow [a x \in F]$  and  
 $\models \{[x \in F]\} \rightarrow [x a \in F],$

- 3) For any  $x, b \in R$ ,

$$\models \{[x + b \in F] \wedge [y + b \in F]\} \rightarrow [x \in F],$$

Obviously, inter-valued fuzzifying k-ideal and Ordinary interval-valued fuzzy k-ideals are equivalent. Therefore, there is no difference between k-ideals.

Now, we have the concept of interval-valued  $\tilde{t}$ -tautology, In fact  $\models_{\tilde{t}} P$  if and only if  $[P] \geq \tilde{t}$ , for all valuations.

We extend the concept of **implication –based fuzzy k-ideals**.

# DEFINITION:

Let  $F$  be an interval-valued fuzzy set of  $R[0,0] < \tilde{t} \leq [1,1]$ . Then  $F$  is called a  **$\tilde{t}$  –implications –based interval-valued fuzzy k-ideal of  $R$** .

If it satisfies the following conditions:

- 1) For any  $x, y \in R$ ,

$$\models_{\tilde{t}} \{[x \in F] \wedge [y \in F]\} \rightarrow [x+y \in F],$$

- 2) For any  $x, a \in R$ ,  
 $\models_{\tilde{t}} \{[x \in F]\} \rightarrow [a x \in F]$  and  
 $\models_{\tilde{t}} \{[x \in F]\} \rightarrow [x a \in F],$

- 3) For any  $x, b \in R$ ,

# THEOREM

An interval-valued fuzzy set  $F$  of  $R$  is a  $\tilde{t}$ -implication –based interval-valued fuzzy k-ideal of  $R$ .

If for all  $a, x, y \in R$  we have:

- 1)  $I(\widetilde{\mu_F}(x) \wedge \widetilde{\mu_F}(y), \widetilde{\mu_F}(x+y)) \geq \tilde{t}$ ,
- 2)  $I(\widetilde{\mu_F}(x), \widetilde{\mu_F}(a x)) \geq \tilde{t}$  and  $I(\widetilde{\mu_F}(x), \widetilde{\mu_F}(x a)) \geq \tilde{t}$
- 3)  $I(\widetilde{\mu_F}(x+a) \wedge \widetilde{\mu_F}(a), \widetilde{\mu_F}(x)) \geq \tilde{t}$

Where  $I$  is an implication operator.

# PROOF:

(1)  $I(\widetilde{\mu}_F(x) \wedge \widetilde{\mu}_F(y), \widetilde{\mu}_F(x+y)) \geq \tilde{t}$  is obvious.

(2) “ $\Rightarrow$ ”

i)  $I_g(\widetilde{\mu}_F(x) \wedge \widetilde{\mu}_F(y), \widetilde{\mu}_F(x+Y) \geq [0.5, 0.5])$

$$\widetilde{\mu}_F(x+y) \geq \widetilde{\mu}_F(x) \wedge \widetilde{\mu}_F(y) \text{ (or)}$$

$$\widetilde{\mu}_F(y) \geq \widetilde{\mu}_F(x+Y) \geq [0.5, 0.5]$$

$$(\widetilde{\mu}_F(x) \wedge$$

Then

$$(\widetilde{\mu}_F(x) \wedge \widetilde{\mu}_F(y) \geq \widetilde{\mu}_F(x+Y) \geq [0.5, 0.5])$$

“ $\Leftarrow$ ” clearly.

3) similar to proof of (2).

### THEOREM:

- i. Let  $I=I_{gr}$  (Gaines- Rescher.) Then F is an  $[0.5, 0.5]$  –implication –Based interval -valued fuzzy k-ideal with thresholds  $(\tilde{r}=[0,0], \tilde{s}=[1,1])$  of R.
- ii. Let  $I=I_g$  (Godel). Then F is an  $[0.5, 0.5]$ -implication –based interval-valued fuzzy k-ideal of R if and only if F is an interval-valued fuzzy k-ideal with thresholds  $(\tilde{r}=[0,0], \tilde{s}=[0.5, 0.5])$  of R.
- iii. Let  $I=I_{cg}$  (contraposition of Godel). Then F is an  $[0.5, 0.5]$ -implication-based interval-valued fuzzy k-ideal of R if and only if F is an interval-valued fuzzy k-ideal with thresholds  $(\tilde{r}=[0.5, 0.5], \tilde{s}=[1,1])$  of R.

### CONCLUSION:

We computed the concepts of generalized fuzzy k-ideal of semi ring with interval-valued membership functions. Also using this idea of quasi – coincide of a fuzzy point with a fuzzy set, the concept of an  $(\alpha, \beta)$  – fuzzy ideal, which is generalization of a fuzzy ideal, in a semi ring is introduced and relationship between logical implication operators and fuzzy k – ideals with thresholds are investigated.

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