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A Study on Fuzzy Graph Theory and Its Applications

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ABSTRACT

Graph theory is one of the parts of current Mathshaving encountered a most noteworthy advancement lately. The theory of fuzzy graphs was created by *Rosenfeld (1975)* in the year 1975. During a similar *time Yeh* and *Bang (1975)* have additionally presented different connectedness ideas in fuzzy graphs. Fuzzy set theory gives us not just an important and ground-breaking portrayal of measurement of vulnerabilities, yet an increasingly reasonable portrayal of dubious ideas communicated in natural languages. The scientific implanting of ordinary set theory into fuzzy has becomes a natural marvel. Therefore, the possibility of fluffiness is an improving one. This Research study analyzes the distance total concept that is a measurement, in a fuzzy graph is presented.

KEYWORDS: Fuzzy Graph, eccentric, peripheral, central nodes, etc.

I. INTRODUCTION

Fuzzy set theory gives significant and ground-breaking portrayal of estimation of vulnerabilities, just as dubious ideas communicated in characteristic dialects.

Graph theory is a significant device to speak to numerous true issues. For instance, an informal community might be spoken to as a graph where vertices speak to accounts (establishments, people, and so on.) and edges speak to the connection between the records. On the off chance that the relations among the records are to be estimated as fortunate or unfortunate as indicated by the recurrence of contacts among the records, fuzzyness ought to be added to portrayal.

Fuzzy set theory offers speculations of set theoretic ideas, for example, crossing point and association. In this manner it carries all out ideas into the dimensional domain.

II IMPORTANT STUDIES

K. R. Bhutani (2003)presented the idea of complete fuzzy graphs and inferred that a total fuzzy graph has no cut nodes. The idea of solid curve in greatest crossing tree and its application in group examination and neural networks were concentrated by Sameera and Sunitha (2006).

P. Bhattacharya (1987) talked about certain properties of fuzzy graphs and presented the thought of erraticism and focus in fuzzy graphs.

A book by **Frank Harary** distributed in 1969 was tremendously famous and empowered mathematicians, physicists, electrical specialists and social researchers to converse with one another.

Rosenfeld (1979) considered fuzzy relations on fuzzy sets and built up the structure of fuzzy graphs, getting analogs of a few graph hypothetical ideas. He presented and analyzed such ideas as connectedness, ways, and extensions, groups, woodlands, cut vertices, and trees.

The idea of control in fuzzy graphs was examined by A. Somasundaram and S. Somasundaram. **A. Somasundaram (1998)** introduced the ideas of autonomous control, all out mastery, associated mastery and mastery in cartesian items and organization of fuzzy graphs. They additionally talked about mastery in fuzzy graph utilizing powerful edges.

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the geodetic number Of legitimate interim graphs can be processed in Polynomial Time. Polynomial Time algorithm is acquainted with register the geodetic number of square prickly plant graphs and a Polynomial Time algorithm to estimate the geodetic number of bipartite change graphs. It is additionally demonstrated that figuring the geodetic number on chordal graphs, bipartite graphs, co-graphs, split graphs and Ptolemaic graph is NP-hard.

III. GRAPH THEORY APPLICATIONS

Graph hypothetical ideas are comprehensively used to play out the different ideas and applications in various territories. Graph theory bargains various applications like image segmentation, data mining, clustering, group theory, coding theory, image capturing, and so on. So also utilizing graph ideas, displaying of network topologies should be possible. In software engineering, graphs are utilized to speak to networks of computational gadgets, correspondences, the progression of calculation and data association and so forth. Additionally in graph theory, circuits, paths and walks are having amazing applications in resource networking, database plan ideas and voyaging sales rep issue. In the comparable manner the most huge thought of graph shading is misused in resource allotment, booking. This prompts the upgrade of part of new algorithms and theorems which has been utilized in amazing applications. It is significant and educational when taking a gander at the spreading of disease, parasites and to get familiar with the effect of excursion that influence different species. Graph theory is likewise broadly utilized in science and safeguarding endeavors in which where the vertex play out the areas, where the genuine species exist and the edges play out the excursion in the middle of the districts. Graph hypothetical ideas are much of the time utilized in activities look into.

The most well-known and successful applications of networks in activities inquire about is booking and arranging of tremendous complex tasks. One of the best notable issues in tasks examine are "CPM (Critical Path Method) and PERT (Project Evaluation Review Technique)". In organic networks, graph theory are utilized as a demonstrating instrument that allows the misuse of most broad graphical invariants along these lines, it is probably going to decide optional Ribonucleic corrosive (RNA) plans numerically. This graphical invariants are deviations of the mastery number of a graph. By the examination the outcomes are worked out to clarify that the deviations of the control number which are utilized to recognize the trees that demonstrate its own structures and it is beyond the realm of imagination to expect to speak to RNA.

IV. PRELIMINARIES

Definition 1: A fuzzy graph is a function's pair σ: $V \rightarrow [0,1]$ and $\mu: V \times V \rightarrow [0,1]$, where for all $u, v \; in \; V$ we have $\mu\,(u,v) \leq \sigma\,(u) \, \Lambda \, \sigma\,(v)$.

Definition 2: Graph wherein the edges are unordered vertex pair is called an undirected graph. A graph wherein the edges are requested vertex pair is known as a coordinated graph. Thus if there is an edge from $v_i$to v_j in G then$(v_i, v_j) \in E$.

Definition 3: Let G: (σ, μ) be a fuzzy graph. The degree of a vertex u is $d_G(u) = d(u) = \sum u \neq v \, \mu(u,v)$.

Definition 4: A graph G is a couple of set (V, E), meant by $G = (V, E)$,whereE is a set of edges& V is a set of vertices . Each edge in E is a couple of vertices in V. Each edge is related with a set comprising of it is possible that a couple of vertices called its end focuses.

Definition 5: Let X be nonempty set.. A fuzzy set A in X is defined as $A = \{(x, \mu A\,(x)) \,/ x \in X\}$which is described by a participation workμ$A\,(x)$: $X \rightarrow [0,1]$.Fuzzy set is an assortment of articles with evaluated enrollment for example having degrees of enrollment.

Definition 6: Let $G = (V, \sigma, \mu)$ be fuzzy graph with $|V| = n$ and $\mu = \{e_1, e_2, \ldots, e_m\}$. If $m_i = \mu(e_i)$is the quality of the connection related with the ith edge, then

$$\sqrt{2\sum_{i=1}^{m} m_i^2 + n(n-1)\,|A|^{\frac{2}{n}}} \leq E(G) \leq \sqrt{2\left(\sum_{i=1}^{m} m_i^2\right)n}.$$

Definition 7: Let $G = (V, \sigma, \mu)$be fuzzy graph and A be its contiguousness matrix. The vitality of G is characterized as the whole of the total of the Eigen values of A.

Definition 8: Let $G = (V, \sigma, \mu)$ be fuzzy graph and A be its contiguousness matrix. The Eigen estimations of A are called Eigen estimations of G. The range of A is known as the range of G. It is meant by Spec G.

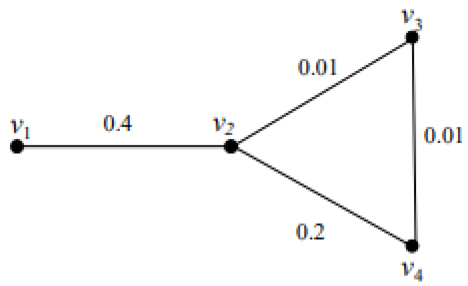
Definition 9: Let $G: (\sigma, \mu)$ is fuzzy graph of the graph $G: (V, E)$, then the distance $d[\sigma(v_i), \sigma(v_j)]$between two of its vertices σ vi and σ vj is the length of shortest path between them, i.e. $d[\sigma\ (v), \sigma\ (v_i)] = Min[\sum_{i,j\in\wedge} \mu(u_i, v_j)$.

V. FUZZY GRAPH STRONG SUM DISTANCE

We attempted another thought for fuzzy distance two naming graph with appropriate clarification and the ideas of limit nodes and inside nodes in fuzzy graph dependent on solid whole distance are presented.

The distance $d[\sigma(vi\ ), \sigma(vj\ )]$ between two nodes $\sigma(vi\ )\ and\ \sigma(vj)$ in a fuzzy graph as the length of the briefest way between them, i.e. $d[\sigma(vi\ ), \sigma(vj)] = Min[\Sigma i, j \in$ μ$(vi\ , vj)]$. Be that as it may, this definition doesn't fulfill the triangle imbalance.

Fuzzy graph



Let $G: (V, \sigma, \mu)$ be a connected fuzzy graph. For any path $P: u_0 - u_1 - u_2 - u_3 - \cdots . - u_n$, length of P is defined as the weights total of the arcs in P i.e. $L(P) = \sum_{i=1}^{n} \mu\ (\mu_{i-1}, u_i)$. If $n = 0$, define $L(P) = 0$ and for $n \geq 1, L(P) > 0$. for any two nodes $u, v\ in\ G, let\ P = \{\ Pi : \ Pi\ is\ a\ u - v\ path, i = 1,2,3, \dots \}$. the distance total between u and v is defined as $ds(u, v) = Min\ \{L(Pi) : \ Pi \in P, i = 1,2,3, \dots \}$.

Theorem 1: In a fuzzy graph $G: (V, \sigma, \mu), d_s: V \times V \to [0,1] is\ a\ metric\ on\ V. i. e. \forall u, v, w \in V$.

- ✓ $d_s(u, v) = 0\ if\ and\ only\ if\ u = v$
- ✓ $d_s(u, v) \geq 0 \forall u, v \in V$
- ✓ $d_s(u, w) \leq d_s(u, v) + d_s(v, w)$
- ✓ $d_s(u, v) = d_s(v, u)$

Proof: Since reversal of a path from u to v is a path from v to u and vice versa,$d_s(u, v) = d_s(v, u)$Let P_1 be a $u - v$ path such that $d_s(u, v) = L(P_1)\ and\ P_2$ be a $v - w$ path such that $ds(v, w) = L(P_2)$. The path P_1followed by $P_2$is a u − w walk and since every walk contains one path, there exists a $u - w$ path in G whose length is at most $d_s(u, v) + d_s(v, w). Therefore, d_s(u, w) \leq d_s(u, v) + d_s(v, w)$.

Theorem 2: If $C^* = (N, A)$indicate a fundamental crisp circular graph with$|N| = |A| = \alpha$then exactly α− 2 cut nodes happened in the fuzzy distance two marking cycle graph$F(C)^\psi = (\sigma_N^\psi, \mu_A^\psi)$.

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Proof: Let $F(C)^{\psi} = (\sigma_N^{\psi}, \mu_A^{\psi})$is a fuzzy distance two labeling cycle with $C^{\psi} = (N, A)$as an underlying crisp circular graph where$|N| = |A| = \alpha$. We realize that any node in a fuzzy graph characterized a fuzzy cut node of that fuzzy graph if its discharge gives the decrement in the estimation of solidarity of the connectedness for some another joined nodes. This suggests node in a fuzzy graph is a fuzzy cut node for that graph in the event that it fill in as a typical node of any two fuzzy extensions. Exactlyα–1 bridges occurred in each fuzzy distance two labeling cycle graph $F(C)^{\psi} = (\sigma_N^{\psi}, \mu_A^{\psi})$(i.e.) fuzzy distance two labeling cycle graph will have single weakest arc only.

Let $\mu_A^{\psi}(p, q)$denote the allocation of this single weakest arc of $F(C)^{\psi} = (\sigma_N^{\psi}, \mu_A^{\psi})$and let $\sigma_N^{\psi}(p)$and $\sigma_N^{\psi}(p)$ respectively be the assignment of the nodes p and q.

Therefore excluding $\sigma_N^{\psi}(p)$&$\mu_A^{\psi}(p, q)$nodes from the graph $F(C)^{\psi} = (\sigma_N^{\psi}, \mu_A^{\psi})$remaining all α– 2 nodes of $F(C)^{\psi} = (\sigma_N^{\psi}, \mu_A^{\psi})$serve as a common node of two fuzzy bridges of the fuzzy distance two labeling cycle $F(C)^{\psi} = (\sigma_N^{\psi}, \mu_A^{\psi})$. Therefore all these α– 2 nodes of will be cut nodes of $F(C)^{\psi} = (\sigma_N^{\psi}, \mu_A^{\psi})$Hence exactly α– 2 cut nodes appeared in every fuzzy distance two labeling cycle $F(C)^{\psi} = (\sigma_N^{\psi}, \mu_A^{\psi})$.

Remarks: The node in a fuzzy distance two labeling cycle graph $F(C)^{\psi} = (\sigma_N^{\psi}, \mu_A^{\psi})$ is either a cut node or end node.

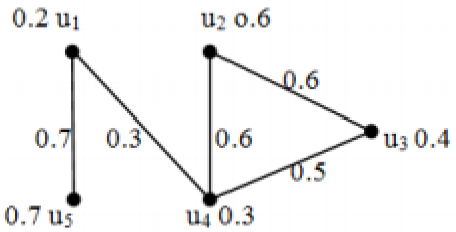
Every fuzzy distance two labeling cycle graph $F(C)^{\psi} = (\sigma_N^{\psi}, \mu_A^{\psi})$has exactly only one weakest arc, say$\sigma_N^{\psi}(p, q)$. Which implies that $\sigma_N^{\psi}(p) and\ \sigma_N^{\psi}(q)$are two end nodes. Hence exactly two end nodes obtained in every fuzzy distance two labeling cycle graph.

Theorem 3: For every 2 strong neighbor's v and u in a connected fuzzy graph $G: (V, \sigma, \mu), |e(u) - e(v)| \le 1$.

Proof: Loss of generality $e(u) \ge e(v)$. Let x be a node farthest from u. i.e. $e(u) = d_{ss}(u, x) \le d_{ss}(u, v) + d_{ss}(v, x)$, by triangle inequality. Therefore $e(u) \le d_{ss}(u, v) + e(v)$, since $e(v) \ge d_{ss}(v, x)$. sincev and u are strong neighbors we have $d_{ss}(u, v) \le 1$.Therefore $e(v) \le 1 + e(v) \Rightarrow 0$

$\le e(u) - e(v) \le 1.$

$\therefore |e(u) - (v)| \le 1.$



Theorem 4: $G_A(\sigma, \mu)$is an anti-fuzzy graph then the distance $d(\sigma(u), \sigma(v))$between two of its vertices $\sigma\ (u)\ and\ \sigma\ (v)$ is the shortest length between them, that is $d(\sigma(u), \sigma(v)) =$ path of $\min[\sum_{u,v\epsilon V} \mu(u, v)]$

In Fig. 9, consider the path from $u_1\ to\ u_3$. Then there exists two path between$\sigma(u_1)\ and\ \sigma(u_3)$. They are

- ✓ $\mu(u1, u4), \mu(u4, u2), \mu(u2, u3)$
- ✓ $\mu(u1, u4), \mu(u4, u3)$

The distance between the vertices $\sigma(u_1),\ and\ (u_3)$

Via path 2, is $\mu(u_1, u_4) + \mu(u_4, u_2) + \mu(u_2, u_3) = 0.3 + 0.6 + 0.6 = 1.5$

Via path 1, is $\mu(u_1, u_4) + \mu(u_4, u_3) = 0.3 + 0.5 = 0.8$

Therefore, $d(\ \sigma(u), \sigma(v)\ ) = min$ [0.8 , 1.5]

Theorem 5: For a graph $G^*$of order P, the antipodal graph $A(G^*) = G^*$if and only if $G^* = K_p$.

Proof: If $G^*$is a non-complete graph of order P, then $A(G^*) \subset \overline{G^*}$,for a graph G^*, antipodal graph $A(G^*) = G^*$ if and only if (a) $G^*$is of diameter 2 or (b) $G^*$is disconnected and the components of $G^*$is an antipodal graph if and just on the off chance that it is the antipodal graph of its supplement. Median Of a graph$G^*$is the set of all vertices v of $G^*$for which the value $d_G^*(v)$is minimized. A graph $G^*$is self-median if and only if the value $d_G^*(v)$is constant over all vertices v of G^*. the status, or on the other hand distance total, of a given vertex v in a graph is characterized by $S(v) = \sum_{v \neq u} d(u, v)$where d (u, v) is distance from a vertex u to v. In other words, a self-median graph G^*G is one in which all the nodes have the same status$S(v)$.

The graphs $C_n, K_{n,n}$ and $K_n$are self-median The status of a vertex $v_i$is denoted by $S(v_i)$and is defined as $S(v_i) = \sum_{v_i \in V} \delta(v_i, v_j)$.The total status of a fuzzy graph $G^*$is denoted by $t[S(G^*)]$ and is defined as $t[S(G^*)] = \sum_{v_i \in V} S(v_i)$.The median of a fuzzy graph G^*meant, is the set of nodes with least status. A fuzzy graphG^*iis said to act elf - median if all the vertices have a similar status. By a fuzzy subset μ on a set X is mean a map $\mu: X \to [0, 1]$.A map $\upsilon: X \times X \to [1, 0]$ is called a fuzzy relation on X if $\upsilon(x, y) \leq min\,(\mu(x,)\, \mu(y))$ $for\ all\ x, y \in X$. A fuzzy relation υ is symmetric if $\upsilon(x, y) = \upsilon(y, x)$ $for\ all\ x, y \in X$.

Theorem 6: Let $G: (\sigma, \mu)$ be an associated fuzzy graph. The askew components of the maximum max creation of the fuzzy G matrix of distance with itself are the fuzzy unconventionalities of nodes.

Proof: Let $d_f = (d_i, j)$be the fuzzyGmatrix of distance.

Then, $d_i, j) = d_f(v_i, v_j)$.In the max-max composition, $D_f o D_f$, the i[th] diagonal entry $d_i, i = \max\begin{Bmatrix} \max(d_i 1, d_1, i)\,, \max(d_i 2, d_2, i)\,, \\ max\,(d_i 3, d_3, i), \ldots, \max(d_i n, d_n, i) \end{Bmatrix} = \max\{d_i, 1, d_i 2, d_i 3, \ldots., d_i n\} = \max\begin{Bmatrix} d_f(v_i, v_1), d_f(v_i, v_2), d_f(v_i, v_3), \\ \ldots, d_f(v_i, v_n) \end{Bmatrix} = e_f(v_i)$. This illustrates the theoremproof.

Theorem 7: Leave G alone a fuzzy tree. At that point v is a g-unpredictable hub of G if and just if v is g-peripheral Gnode.

Next, with respect to the δ - distance and ss – distance it is established that every connected fuzzy graph $G: (\sigma, \mu)$ is self-centered.

Theorem 8: A connected fuzzy graph $G: (\sigma, \mu)$is fuzzy conceited if and just if all the passages in the foremost askew of the maximum max formation of the fuzzy distance matrix with itself are the equivalent.

Proof: As demonstrated in hypothesis 6, the essential askew passages in the maximum - max organization of the fuzzy distance matrix with itself are the fuzzy unconventionalities of the nodes. In the event that they are same, this implies$e_f(u)$is the same for all u in G. Then G is fuzzy self-centered. Hence the proof is completed.

VI. CONCLUSION

It is realized that fuzzy graph theory has various applications in present day science and engineering. We genuinely trust its wide going applications of graph theory and the interdisciplinary idea of fuzzy set theory, if appropriately mixed together could clear a route for a considerable development of fuzzy graph theory. The idea of fuzzy distance is significant as it speaks to the net stream between a given pair of nodes of a fuzzy graph. The idea of fuzzy focus, fuzzy self-centered graphs, and supplement of a fuzzy graph are additionally presented. The investigation of fuzzy graphs made right now a long way from being finished. Distance is a significant idea in the whole graph theory. Right now, certifiable exertion is made to sum up the idea of distance.

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