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Co-Efficient of Range Labeling for Some Trees

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Abstract: In this paper, we focus on one type of labeling is called co-efficient of range labeling, we have introduced co-efficient of range labeling for Double star graph, Star trees,Spider Trees and Symmetrical Trees.

Keywords: Labeling, Graceful Labeling, Range labeling,Co-effiecient of Range labelling,Double star graph,Star Trees,Spider Trees,Symmetrical Trees.

Introduction: In this paper study for each graph finite and simple. A graph G=(S, T) if S is a vertices and T is a edges. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions , First introduced labeling was Rosa in 1967(9). In this article studied for new labeling is co-efficient of Range labeling for some Trees. The first introduced Range labeling for some graphs by R. Jahir Hussain and J. Senthamizh Selvan motivated by Range labeling we have developed co-efficient of Range labeling.

1.Preliminary

Definition 1.1:Range labeling:

Let G=(S, T) be a graph with n vertices. A bijection on $\propto$: $S\rightarrow\{1,2,...,n\}$ is called a range labeling if for each edge T is

$\propto^*(T)$= Maximum value $(S_{k,}S_{k+1})$ – Minimum value $(S_{k,}S_{k+1})$

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Definition 1.2:Graceful labelling(7):

A graceful labeling of a graph G is a vertex labeling $\propto$: S→[0,m] such that $\propto$ is injective and the edge labeling $\propto$: T→[1,m] is defined by $\propto^*[S_k\ S_{k+1}] = |\propto(S_k) - \propto(S_{k+1})|$is also injective. If a graph G admits a graceful labeling. We say G is a graceful graph.

Definition 1.3: Coefficient of Range:

$$= \frac{\text{maximum value} - \text{minimum value}}{\text{maximum value} + \text{minimum value}}$$

Definition 1.4 : Star Trees(1):

A tree wih one internal node and K leaves is said to be star S_1,k that happen to be a complete by part a graph k_1,k

Definition 1.5: Spider Trees(1):

A Spider tree (or) Spider graph is a tree with at most one vertex of degree greater than 2. If such a vertex exists. It is called the tree branch point of the tree. A leg of a Spider tree is any one of the parts from the branch points to a leaf of the tree.

Definition 1.6:Symmetrical tree(7):

A rooted tree in which every level contains vertices of the same degree is called Symmetrical tree.

2.Main results

The idea of graceful labeling and Range labeling motivate us to decide study for new labeling is Co-efficient of Range labeling.

Definition 2.1: Co-efficient of Range Labeling:

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Let G=(S, T) be a graph with m vertices. A function from β: S→ {1, 2... m} is called a co-efficient of range labeling if for every edge T is distinct and T is defined by.

$\beta^*(T)$=Numerator decrease in the difference of (S_r, S_{r+1})

$$\left[\frac{\text{Maximum value}(S_r, S_{r+1}) - \text{Minimum value}(S_r, S_{r+1})}{\text{Maximum value}(S_r, S_{r+1}) + \text{Minimum value}(S_r, S_{r+1})} + 3 \text{ minimum value}(S_r, S_{r+1})\right]$$

Theorem 2.2: The Double Star graph is a accept co-efficient of Range labeling.

Proof:

Let G=(S, T) be a double star graph. It is fixed by $S_{m,n}$ and S_1, S_2 are not pendent vertices in $S_{m,n}$. Let r_i's m pendent vertices to S_1 and r_j's n pendent vertices to S_2.

If β: S→ {1,2...m} & β′: R→ {1,2...n}

$\beta^*(T)$=Numerator decrease in the difference of (S_1, S_2)

$$\left[\frac{\text{Maximum value}(S_1, S_2) - \text{Minimum value}(S_1, S_2)}{\text{Maximum value}(S_1, S_2) + \text{Minimum value}(S_1, S_2)} + 3 \text{ Minimum}(S_1, S_2)\right]$$

If S_1 is a maximum, S_2 is a minimum.

$\beta^*(T)$=Numerator decrease in the difference of $(S_1, S_2)\left[\frac{S_1 - S_2}{S_1 + S_2} + 3\, S_2\right]$

=Numerator decrease in the difference of $(S_1, S_2)\left[\frac{S_1 - S_2 + 3S_1(S_1 + S_2)}{S_1 + S_2}\right]$

$=\frac{3S_2(S_1+S_2)}{(S_1+S_2)}$

$= 3S_2$

If suppose S_2 is a maximum, S_1 is a minimum.

$\beta^*(T)$=Numerator decrease in the difference of $(S_1, S_2)\left[\frac{S_2 - S_1}{S_2 + S_1} + 3\, S_2\right]$

=Numerator decrease in the difference of $(S_1, S_2)\left[\frac{S_2 - S_1 + 3S_1(S_2 + S_1)}{S_2 + S_1}\right]$

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$= 3S_1$

β^*(T)=Numerator decrease in the difference of (S_1, r_i)

$$\left[\frac{\text{Maximum value}(S_1, r_i) - \text{Minimum value}(S_1, r_i)}{\text{Maximum value}(S_1, r_i) + \text{Minimum value}(S_1, r_i)} + 3\ \text{Minimum}(S_1, r_i)\right]$$

If S_1 is a maximum, r_i is a minimum.

β^*(T)=Numerator decrease in the difference of $(S_1, r_i)\left[\frac{S_1 - r_i}{S_1 + r_i} + 3\ r_i\right]$

=Numerator decrease in the difference of $(S_1, r_i)\left[\frac{S_1 - r_i + 3r_i(S_1 + r_i)}{S_1 + r_i}\right]$

$= \frac{3r_i(S_1 + r_i)}{(S_1 + r_i)}$

$= 3r_i$

If suppose r_i is a maximum, S_1 is a minimum.

β^*(T)=Numerator decrease in the difference of $(S_1, r_i)\left[\frac{r_i - S_1}{r_i + S_1} + 3\ S_1\right]$

=Numerator decrease in the difference of $(S_1, r_i)\left[\frac{r_i - S_1 + 3S_1(r_i + S_1)}{r_i + S_1}\right]$

$= 3S_1$

β^*(T)=Numerator decrease in the difference of (S_2, r_j)

$$\left[\frac{\text{Maximum value}(S_2, r_j) - \text{Minimum value}(S_2, r_j)}{\text{Maximum value}(S_2, r_j) + \text{Minimum value}(S_2, r_j)} + 3\ \text{Minimum}(S_2, r_j)\right]$$

If S_2 is a maximum, r_j is a minimum.

β^*(T)=Numerator decrease in the difference of $(S_2, r_j)\left[\frac{S_2 - r_j}{S_2 + r_j} + 3\ r_j\right]$

=Numerator decrease in the difference of $(S_2, r_j)\left[\frac{S_2 - r_j + 3r_j(S_2 + r_j)}{S_2 + r_j}\right]$

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$=\frac{3r_j(S_2+r_j)}{(S_2+r_j)}$

$= 3r_j$

If suppose r_j is maximum, S_2 is a minimum.

$\beta^*(T)$=Numerator decrease in the difference of $(S_2 , r_j)\left[\frac{r_j-S_2}{r_j+S_2}+3\ S_2\right]$

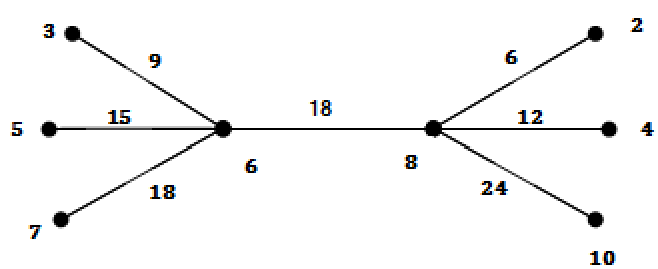
=Numerator decrease in the difference of $(S_2 , r_j)\left[\frac{r_j-S_2+3S_2(r_j+S_2)}{r_j+S_2}\right]$

$= 3S_2$

Hence, every double step graph received co-efficient of range labeling.

∴ All double star graph is a co-efficient of Range graph.

Example 1:



Theorem 2.3: Every Star tree is a accept co-efficient of Range Labeling.

Proof :

Let G = (S, T) be a graph with S_1 is a internal node and 8 leafes.

Let β: S⟶{1,2,. . .m}

β^* (T)= Numerator decrease in the difference of(S_1,S_2)

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$$\left[\frac{\text{maximum value}(s_1,s_2)-\text{minimum value }(s_1,s_2)}{\text{maximum value}(s_1,s_2)+\text{minimum value}(s_1,s_2)}+3\text{ minimum value}(s_1,s_2)\right]$$

If S_1 is a minimum value, S_2 is a maximum value

$\beta^*(T)$ = Numerator decrease in the difference of $(S_1,S_2)\left[\frac{s_2-s_1}{s_2+s_1}+3s_1\right]$

= Numerator decrease in the difference of (S_1,S_2) $\left[\frac{s_2-s_1+3s_1(s_2+s_1)}{s_2+s_1}\right]$

$= 3s_1$

If S_2 is a minimum value, S_1 is a maximum value

$\beta^*(T)$ = Numerator decrease in the difference of $(S_1,S_2)\left[\frac{s_1-s_2}{s_1+s_2}+3s_2\right]$

= Numerator decrease in the difference of (S_1,S_2) $\left[\frac{s_1-s_2+3s_2(s_1+s_2)}{s_1+s_2}\right]$

$= 3s_2$

β^* (T)= Numerator decrease in the difference of (S_1,S_3)

$$\left[\frac{\text{maximum value}(s_1,s_3)-\text{minimum value }(s_1,s_3)}{\text{maximum value}(s_1,s_3)+\text{minimum value}(s_1,s_3)}+3\text{ minimum value}(s_1,s_3)\right]$$

If S_1 is a minimum value, S_3 is a maximum value

$\beta^*(T)$ = Numerator decrease in the difference of $(S_1,S_3)\left[\frac{s_3-s_1}{s_3+s_1}+3s_1\right]$

$= 3s_1$

If S_3 is a minimum value, S_1 is a maximum value

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$\beta^*(T)$ = Numerator decrease in the difference of $(S_1,S_3)\left[\frac{s_1-s_3}{s_1+s_3}+3s_3\right]$

$= 3s_3$

β^* (T)= Numerator decrease in the difference of (S_1,S_4)

$$\left[\frac{\text{maximum value}(s_1,s_4)-\text{minimum value}(s_1,s_4)}{\text{maximum value}(s_1,s_4)+\text{minimum value}(s_1,s_4)}+3\text{ minimum value}(s_1,s_4)\right]$$

If S_4 is a minimum value, S_1 is a maximum value

$\beta^*(T)$ = Numerator decrease in the difference of $(S_1,S_4)\left[\frac{s_1-s_4}{s_1+s_4}+3s_4\right]$

$= 3s_4$

If S_1 is minimum value, S_4 is a maximum value

$\beta^*(T)$ = Numerator decrease in the difference of $(S_1,S_4)\left[\frac{s_4-s_1}{s_4+s_1}+3s_1\right]$

$= 3s_1$

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β^*(T)=Numerator decrease in the difference of (s_1,s_9)

$$\left[\frac{\text{maximum value}(s_1,s_9)-\text{minimum value}(s_1,s_9)}{\text{maximum value}(s_1,s_9)+\text{minimum value}(s_1,s_9)}+3\text{ minimum value}(s_1,s_9)\right]$$

If s_1 is a minimum , s_9 is a maximum

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$$\beta^*(T)=\text{Numerator decrease in the difference of } (s_1, s_9)\left[\frac{s_9-s_1}{s_9+s_1}+3s_1\right]$$

$$=3s_1$$

If s_9 is a minimum , s_1 is a maximum

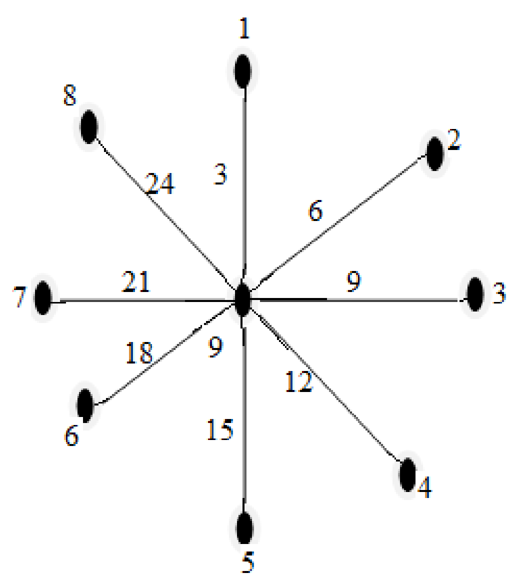
$$\beta^*(T)=\text{Numerator decrease in the difference of } (s_1, s_9)\left[\frac{s_1-s_9}{s_1-s_9}+3s_9\right]$$

$$=3s_9$$

Hence, Every Star tree graph received Co-efficient of Range labelling.

Therefore, All Star tree graph is a Co-efficient of Range graph

Example 2:



Theorem:2.4 Every Spider tree is a co-efficient of Range Labeling.

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Proof.

Let G = (S, T) be a graph

Let β: S → {1,2, . . . m}.

A Spider is a tree with at most one vertex of degree greater than 2 and this vertex is called the branch vertex and is denoted by S_0. A leg of a Spider graph is a path from the branch vertex to a leaf of the tree. The leaf is denoted by S_1,S_2,S_3.

β^* (T)= Numerator decrease in the difference of (S_0, S_1)

$$\left[\frac{\text{maximum value}(s_0,s_1) - \text{minimum value}(s_0,s_1)}{\text{maximum value}(s_0,s_1) + \text{minimum value}(s_0,s_1)} + 3\ \text{minimum value}(s_0,s_1)\right]$$

If S_1 is a minimum value, S_0 is a maximum value

$\beta^*(T)$ = Numerator decrease in the difference of $(S_0, S_1)\left[\frac{s_0 - s_1}{s_0 + s_1} + 3s_1\right]$

$= 3s_1$

If S_0 is a minimum value, S_1 is a maximum value

$\beta^*(T)$ = Numerator decrease in the difference of $(S_0, S_1)\left[\frac{s_1 - s_0}{s_1 + s_0} + 3s_0\right]$

$= 3s_0$

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β^* (T)= Numerator decrease in the difference of (S_0, S_3)

$$\left[\frac{\text{maximum value}(s_0,s_3) - \text{minimum value}(s_0,s_3)}{\text{maximum value}(s_0,s_3) + \text{minimum value}(s_0,s_3)} + 3\ \text{minimum value}(s_0,s_3)\right]$$

If S_3 is a minimum value, S_0 is a maximum value

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$\beta^*(T)$ = Numerator decrease in the difference of $(S_0, S_3)\left[\frac{s_0-s_3}{s_0+s_3}+3s_3\right]$

= $3s_3$

If S_0 is minimum value, S_3 is a maximum value

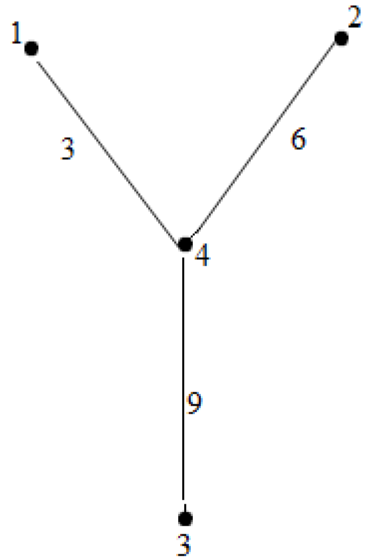
$\beta^*(T)$ = Numerator decrease in the difference of $(S_0, S_3)\left[\frac{s_3-s_0}{s_3+s_0}+3s_0\right]$

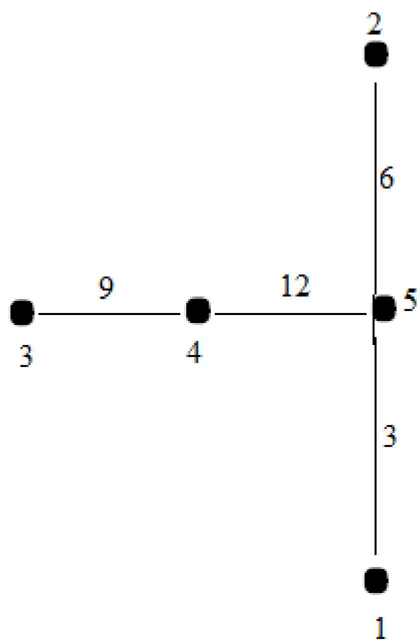
= $3s_0$

Hence, Every Spider tree graph received co-efficient of range labelling.

Therefore, All Spider tree graph is a co-efficient of range graph.

Example 3:





Theorem:2.5

The symmetrical tree is a accept co-efficient of Range labelling

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Proof:

Let G=(S,T) be a group

Let $\beta: S_1 \rightarrow \{1,2,\ldots,m\}$, $\beta_1: S_{1i} \rightarrow \{1,2,\ldots,m\}$ for i=1,2

$\beta: S_{ij} \rightarrow \{1,2,\ldots,m\}$ for i,j =1,2

β^* (T)= Numerator decrease in the difference of(S_1,S_{1i})

$$\left[\frac{\text{Maximum value }(S_1,S_{1i}) - \text{Minimum value}(S_1,S_{1i})}{\text{Maximum value }(S_1,S_{1i}) + \text{Minimum value}(S_1,S_{1i})} + 3\text{ minimum value}(S_1,S_{1i})\right]$$

If S_1 is a maximum, S_{1i} is a minimum

β^* (T)= Numerator decrease in the difference$(S_1,S_{1i})\left[\frac{S_1 - S_{1i}}{S_1 + S_{1i}} + 3S_1\right]$

= Numerator decrease in the difference$(S_1,S_{1i})\left[\frac{S_1 - S_{1i} + 3(S_{1i}(S_1 + S_{1i})}{S_1 + S_{1i}}\right]$

$= \frac{3(S_{1i})(S_1 + S_{1i})}{S_1 + S_{1i}}$

$= 3S_{1i}$

If S_{1i} is a maximum, S_1 is a minimum

β^* (T)= Numerator decrease in the difference$(S_1,S_{1i})\left[\frac{S_1 - S_{1i}}{S_1 + S_{1i}} + 3S_1\right]$

= Numerator decrease in the difference$(S_1,S_{1i})\left[\frac{S_{1i} - S_1 + 3\,S_1\,(S_{1i} + S_1)}{S_{1i} + S_1}\right]$

β^* (T)$=\frac{3\,S_1(S_{1i} + S_1)}{(S_{1i} + S_1)}$

β^* (T) = $3S_1$

$\beta_1^*(T)$ Numerator decrease in the difference $(S_{1i} + S_{ij})$

$$\frac{\text{Maximum value }(S_{1i}, S_{ij}) - \text{Minimum value}(S_{1i}, S_{ij})}{\text{Maximum value }(S_{1i}, S_{ij}) + \text{Minimum value}(S_{1i}, S_{ij})} + 3\text{ minimum value}(S_{1i}, S_{ij})$$

If S_{1i} is maximum, S_{ij} is a minimum

$$\beta_1^*(T) = \text{Numerator decrease in the difference}(S_{1i}, S_{ij})\left[\frac{S_{1i} - S_{ij}}{S_{1i} + S_{ij}} + 3S_{1i}\right]$$

$$= \text{Numerator decrease in the difference}(S_{1i}, S_{ij})\left[\frac{S_{1i} - S_{ij} + 3S_{1i}(S_{1i} + S_{ij})}{S_{1i} + S_{ij}}\right]$$

$$= \frac{3S_{1i}(S_{1i} + S_{ij})}{S_{1i} + S_{ij}}$$

$$\beta_1^*(T) = 3S_{1i}$$

If take S_{ij} is maximum, S_{1i} is minimum

$$\beta_1^*(T) = \text{Numerator decrease in the difference}(S_{1i}, S_{ij})\left[\frac{S_{ij} - S_{1i}}{S_{ij} + S_{1i}} + 3S_{ij}\right]$$

$$= \text{Numerator decrease in the difference}(S_{1i}, S_{ij})\left[\frac{S_{ij} - S_{1i} + 3S_{ij}(S_{ij} + S_{1i})}{S_{ij} + S_{1i}}\right]$$

$$= \frac{3S_{ij}(S_{ij} + S_{1i})}{S_{ij} + S_{1i}}$$

$$= 3S_{ij}$$

Hence, Every Symmetrical tree graph received Co-efficient of Range labeling

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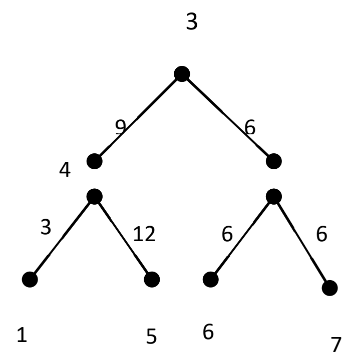
∴ All Symmetrical tree graph is a Co-efficient of Range graph.

Example 4:



Conclusion :

In this paper we have discussed a few graphs received coefficient of Range labelling. Further analysis on coefficient of Range labeling may receive for some special graphs.

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