JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 3708-3713
https://publishoa.com
ISSN: 1309-3452

Γ-(*λ* , *δ*)-N-Derivation on Prime *Γ*-Near-Ring

Hiba A. Ahmed
Baghdad University
College of Science
Department of Mathematics
Baghdad
Iraq
*E-mail: **hiba.ahmed@sc.uobaghdad.edu.iq**

Received: 2022 March 15; **Revised:** 2022 April 20; **Accepted:** 2022 May 10

Abstract

This paper's primary goal is to define μ-(λ , δ)-m-derivation in μ-near-ring Nand investigate a few properties.We also research and debate the commutativity of addition multiplication of prime μ-near-ring with μ-(λ , δ)-m-derivation.

Subject Classification:*Primary 93A30, Secondary 49K15*

Keywords:μ-(λ , δ)-m-derivation, prime μ-near-ring.

1. Introduction

Throughout this paper. A $\boldsymbol{\mu}$- near ring were submitted in [1,5,6]. A $\boldsymbol{\mu}$- near- ring N is called prime $\boldsymbol{\mu}$-near-ring if N has the property that for s,r$\in$ N, $\boldsymbol{\mu}$N$\boldsymbol{\mu}$r =0 implies s= 0 or r=0 [6,7].The other commutators are; $[s,r]_\rho$= sρr – rρs and (s, r) = s+r–s–r denote the additive-group commutator [4,7]. $\boldsymbol{\mu}$-near-ring N is called commutative if (N, +) is abelian [1,5,6]. Inspired by these concepts, Ashraf [2] introduced (σ,τ)-n-derivation in near-rings and studied its various properties.

An n-additive mapping $h{:}\underbrace{N \times N \times \ldots \times N}_{n-times} \longrightarrow N$ is called μ-(λ,δ)-m-derivation (μ -(λ,δ)-m-der.) of N if there exist automorphism mappings $\lambda, \delta{:}\ N \longrightarrow N$ such that the relations

$$h(k_1\gamma k_1{}', k_2, \dots, k_m) = h(k_1, k_2, \dots, k_m)\gamma\lambda\left(k_1{}'\right) + \delta(k_1)\gamma h(k_1{}', k_2, \dots, k_m)$$

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 3708-3713
https://publishoa.com
ISSN: 1309-3452

$h(k_1, k_2\gamma {k_2}', \dots, k_m) = h(k_1, k_2, \dots, k_m)\gamma\lambda({k_2}') + \delta(k_2)\gamma h(k_1, {k_2}', \dots, k_m)$ ⋮

$h\left(k_1, k_2, \dots, k_n\gamma {k_m}'\right) = h(k_1, k_2, \dots, k_m)\gamma\lambda\left({k_m}'\right) + \delta(k_m)\gamma h(k_1\ , k_2, \dots, {k_m}')$

hold for all $k_1, {k_1}', k_2, {k_2}', \dots, k_m, {k_m}' \in N$ and $\gamma \epsilon \mu$.In this work, we defined the conceptof μ-(λ , δ)-n-derivations in μ-near-rings. We will give some important results to μ-(λ , δ)-n-derivations. Ashraf, Ali have proved some results on commutativity of prime near-ring with (σ , τ)-n-derivations. In this paper we will work and study the result in [1,2,3], but in case μ-(λ, δ)-m-derivations we discuss the commutativity of addition and multiplication of prime Γ-near-ring .

2. Preliminaries.

This section starts with the lemmas that are necessary for constructing the proofs of our primary results. We recall prime μ- near ring by μ(N) and μ-(λ , δ)-m-derivation by μ-(λ , δ)-m-der. and automorphism by aut.

Lem. 2.1[1,5]. When$N$ be a $\mu(N)$. There is a component u of Z(N) such that
u + u ∈ Z(N), then (N, +) is abelian.

Lemma 2.2: When$N$ be a μ(N) and h be a μ-(λ , δ)-m-der. of N, then
$(h(k_1, k_2, \dots, k_m)\gamma\lambda(k_1') + \delta(k_1)\gamma h(k_1', k_2, \dots, k_m))\beta y = h(k_1, k_2, \dots, k_m)\gamma\lambda(k_1')\beta y + \delta(k_1)\gamma h(k_1', k_2, \dots, k_m)\beta y$,hold for every $k_1, k_1', k_2, k_2', \dots, k_n, k_m', y \epsilon N$ and $\gamma, \beta \epsilon \mu$.

Proof: For every $k_1, k_1', y, k_2, \dots, k_m \epsilon N$, we have $h\left((k_1\gamma k_1')\beta y, k_2, \dots, k_m\right) = h\left(k_1\gamma k_1', k_2, \dots, k_m\right)\beta\lambda(y) + \delta(k_1\gamma k_1')\beta h(y, k_2, \dots, k_m)$

$= (h(k_1, k_2, \dots, k_m)\gamma\lambda\left(k_1'\right) +$
$\delta(k_1)\gamma h\left(k_1', k_2, \dots, k_m\right))\beta\lambda(y) + \delta(k_1)\gamma\delta(k_1')\beta h(y, k_2, \dots, k_m)$. (1)

Also, $\left(k_1\gamma(k_1'\beta y), k_2, \dots, k_m\right) = h(k_1, k_2, \dots, k_m)\gamma\lambda(k_1'\beta y) + \delta(k_1\ )\gamma h\left(k_1'\beta y, k_2, \dots, k_m\right) =$
$h(k_1, k_2, \dots, k_m)\gamma\lambda\left(k_1'\right)\beta\lambda(y)$
$+ \delta(k_1)\gamma h\left(k_1', k_2, \dots, k_m\right)\beta\lambda(y) + \delta(k_1)\gamma\delta(k_1')\beta h(y, k_2, \dots, k_m)$. (2)

Combining equations (1) and (2), we get
$(h(k_1, k_2, \dots, k_m)\gamma\lambda\left(k_1'\right) + \delta(k_1)\gamma h\left(k_1', k_2, \dots, k_m\right)\beta\lambda(y) =$
$h(k_1, k_2, \dots, k_m)\gamma\lambda\left(k_1'\right)\beta\lambda(y) + \delta(k_1)\gamma h\left(k_1', k_2, \dots, k_m\right)\beta\lambda(y)$

Sinceλ is an aut. we get
$(h(k_1, k_2, \dots, k_m)\gamma\lambda\left(k_1'\right) + \delta(k_1)\gamma h\left(k_1', k_2, \dots, k_m\right)\beta y =$

Volume 13, No. 3, 2022, p. 3708-3713
https://publishoa.com
ISSN: 1309-3452

$h(k_1, k_2, \dots, k_m)\gamma\lambda(k_1')\beta y + \delta(k_1)\gamma h(k_1', k_2, \dots, k_m)\beta y$

Similarly, other (n−1) relations can be proved.

Lemma 2.3: When N be a μ(N), h a non-zero μ-(λ , δ)-m-der. of N and $x \epsilon N$.

(i) If $h(N, N, \dots, N)\gamma x = \{0\}$, then $k = 0$.

(ii) If $x\gamma h(N, N, \dots, N) = \{0\}$, then $k = 0$.

Proof:(i) By supposition have

$h(k_1, k_2, \dots, k_m)\gamma k = 0$, for every $k_1, k_2, \dots, k_m \epsilon N$ and $\gamma \epsilon \mu$. (3)

Taking $k_1\beta y$ in place of k_1, where $y \epsilon N$, in eq. (3), using Lem. (2.2) obtain

$h(k_1, k_2, \dots, k_m)\beta\lambda(y)\gamma k + \delta(k_1)\beta h(y, k_2, \dots, k_m)\gamma k = 0$, for every $k_1, y, k_2, \dots, x_m \epsilon N$ and $\gamma, \beta \epsilon \mu$.

Using eq. (3) again acquire $h(k_1, k_2, \dots, k_m)\beta\lambda(y)\gamma k = 0$.

Where λ is an aut., then we have $h(k_1, k_2, \dots, k_m)\Gamma N\Gamma k = \{0\}$.

Since $h \neq 0$, G *is* $\mu(N)$ obtain $x = 0$. It can be demonstrated in a similar manner.

3. Main result.

Th.3.1: When N be $\mu(N)$ and h_1, h_2 be any two non-zero μ-(λ , δ)-m-der. of N. If $[h_1(N, N, \dots, N), h_2(N, N, \dots, N)]_\gamma = \{0\}$, thus $(N, +)$ is abelian.

Proof: Presume $[h_1(N, N, \dots, N), h_2(N, N, \dots, N)]_\gamma = \{0\}$.

When z and $z + z$ commute element wise with $h_2(N, N, \dots, N)$, thus

$z\gamma h_2(k_1, k_2, \dots, k_m) = h_2(k_1, k_2, \dots, k_m)\gamma z$, for every $k_1, k_2, \dots, k_m \epsilon N, \gamma \epsilon \mu$. (4)

Also, $(z + z)\gamma h_2(k_1, k_2, \dots, k_m) = h_2(k_1, k_2, \dots, k_m)\gamma(z + z)$ (5)

Replacing $k_1 + s$ for k_1 in eq. (5) obtain

$(z + z)\gamma h_2(k_1 + s, k_2, \dots, k_m) = h_2(k_1 + s, k_2, \dots, k_m)\gamma(z + z)$

From eq. (4) and (5) the above eq. get

$z\gamma h_2(k_1 + s - k_1 - s, k_2, \dots, k_m) = 0$. Hence, $z\gamma h_2((k_1, s), k_2, \dots, k_m) = 0$.

Taking $z = h_1(y_1, y_2, \dots, y_m)$ give $h_1(y_1, y_2, \dots, y_m)\gamma h_2((k_1, s), k_2, \dots, k_m) = 0$.

By Lem. (2.3), get $h_2((k_1, s), k_2, \dots, k_m) = 0$, for every $k_1, s, k_2, \dots, k_m \epsilon N$. (6)

Then for every $w \epsilon G$, $w\beta(k_1, s) = w\beta(k_1 + s - k_1 - s) = w\beta k_1 + w\beta s - w\beta k_1 - w\beta s$

$= (w\beta k_1, w\beta s) = w\beta(k_1, s)$

$w\beta(k_1, s)$ is also additive commutator of N. Putting $w\beta(k_1, s)$ instead of additive commutator (k_1, s) in eq. (6) obtain

$h_2(w\beta(k_1, s), k_2, \dots, k_m) = 0$, for every $k_1, s, k_2, \dots, k_m, w \epsilon N$ and $\beta \epsilon \mu$.

Therefore, $h_2(w, k_2, \dots, k_m)\beta\lambda(k_1, s) + \delta(w)\beta h_2((k_1, s), k_2, \dots, k_m) = 0$

Using eq. (4) in previous eq. yields $h_2(w, k_2, \dots, k_m)\beta\lambda(k_1, s) = 0$

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 3708-3713
https://publishoa.com
ISSN: 1309-3452

Since λ is an aut. employ Lem.(2.3) acquire$(k_1, s)= 0$. Then $(N, +)$ is abelain.

Th.3.2: When$N$ be a $\mu(N)$, $h\mu$-(λ, δ)-m-der. of N .If $h(k_1,k_2,...,k_m)\gamma\lambda(y_1)= \delta(k_1)\gamma h(y_1,y_2,...,y_m)$, for every$k_1, k_2,...,k_m,y_1, y_2,...,y_m \epsilon N$ and $\gamma \epsilon \mu$, thus$h = 0$.

Proof: Presume for every$k_1, k_2,..., k_m, y_1, y_2,...,y_m \epsilon N$ and $\gamma \epsilon \mu$.

$h(k_1,k_2,...,k_m)\gamma\lambda(y_1)= \delta(k_1)\gamma h(y_1,y_2,...,y_m)$.(7)

Substituting$y_1\beta z_1$for y_1, where $z_1 \epsilon N$ in eq. (7), acquire

$h(k_1,k_2,...,k_m)\gamma\lambda(y_1)\beta\lambda(z_1)= \delta(k_1)\gamma h(y_1\beta z_1,y_2,...,y_m) =$

$\delta(k_1)\gamma h(y_1,y_2,...,y_m)\beta\lambda(z_1) + \delta(k_1)\gamma\delta(y_1)\beta h(z_1,y_2,...,y_m)$

Employ eq. (7) in above eq. obtains

$\delta(x_1)\gamma\delta(y_1)\beta h(z_1,y_2,...,y_m) = 0$, for every$x_1,z_1,y_1,y_2,...,y_m \epsilon N$, $\gamma , \beta \epsilon \mu$.

Since δ is aut. of N, we get$u\gamma v\beta h(z_1,y_2,...,y_m) = 0$, wherever$u, v \epsilon G$.

Now it's time to replace u by $h(z_1,y_2,...,y_m)$ in pervious eq. obtain

$h(z_1,y_2,...,y_m) \mu N \mu h(z_1,y_2,...,y_m)=\{0\}$.Since N be $\mu(N)$take $h=0$.

Th. 3.3: When$N$ be a $\mu(N)$and ha non-zero μ-(λ , δ)-m-der. of N.If $K = \{a\epsilon N| [f(N, N, . . ., N), \delta(a)]_\gamma = \{0\}\}$, give$a \epsilon K$ implies either $h(a, a, . . ., a) = 0$ or
$a \epsilon Z(N)$.

Proof: We have

$h(k_1, k_2, . . ., k_m)\gamma\delta(a) =\delta(a) \gamma h(k_1, k_2, . . ., k_m)$,

for every$k_1 ,k_2,...,k_m \epsilon N$ and $\gamma \epsilon \mu$. (8)

Taking $a\beta k_1$ in place of x_1 in eq. (8) and utilizing Lem.(2.2) obtains

$h(a, k_2, . . ., k_m)\beta\lambda(k_1)\gamma\delta(a) + \delta(a)\beta h(k_1, k_2, . . ., k_m)\gamma\delta(a) =$

$\delta(a) \gamma h(a, k_2, . . ., k_m)\beta\lambda(k_1) + \delta(a)\gamma\delta(a)\beta h(k_1, k_2, . . ., k_m)$

Employ eq. (8) in above eq. acquire

$$h(a, k_2, . . ., k_m)\beta\lambda(k_1)\gamma\delta(a) = \delta(a) \gamma h(a, k_2, . . ., k_m)\beta\lambda(k_1) \quad (9)$$

Putting $k_1\gamma y_1$for k_1, where $y_1 \epsilon N$ in eq. (9) employit again, procure

$h(a, k_2, . . ., k_m)\beta\lambda(k_1)\gamma[\lambda(y_1), \delta(a)]_\gamma = 0$, for every$k_1, y_1, k_2,...,k_m \epsilon N$ and $\gamma , \beta \epsilon\mu$.

Since λ an aut., we have $h(a, k_2, . . ., x_m) \mu G \mu [\lambda(y_1), \delta(a)]_\gamma = \{0\}$.

Since λ and δ are aut. , G is $\mu(N)$ obtain

Either$a \epsilon Z(N)$or $h(a,k_2, . . .,k_m) = 0$, for all $k_2,...,k_m \epsilon N$ .As a result, in particular

$h(a, a,...,a) = 0$.

Th.(3.4): When$N$ be a $\mu(N)$, h a μ-(λ , δ)-m-der.and $a \epsilon N$.If $[f(N, N, . . ., N), a]_{\lambda,\delta}= \{0\}$, then either $h(a, k_2, . . ., k_m) = 0$ for every$k_2,...,k_m \epsilon N$ or $a \epsilon Z(N)$.

Proof: Suppose that

$h(k_1, k_2, \ldots, k_m)\gamma\lambda\ (a) = \delta(a)\gamma h(k_1, k_2, \ldots, k_m)$, for every $k_1, k_2, \ldots, k_m \epsilon N$, $\gamma \epsilon \mu$ (10)

Taking $a\beta x_1$ in place of x_1 in eq. (10) also takingLem. (2.2) obtains

$h(a, k_2, \ldots, k_m)\beta\lambda(k_1)\gamma\lambda\ (a) + \delta(a)\beta h(k_1, k_2, \ldots, k_m)\gamma\lambda\ (a) =$

$\delta(a)\ \gamma h(a, k_2, \ldots, k_m)\beta\lambda(k_1) + \delta(a)\gamma\delta\ (a)\beta h(k_1, k_2, \ldots, k_m)$.

Take eq. (10) in above eq. obtains

$h(a, k_2, \ldots, k_m)\beta\lambda(k_1)\gamma\lambda\ (a) = \delta(a)\gamma h(a, k_2, \ldots, k_m)\beta\lambda(k_1)$ (11)

Putting $k_1\gamma y_1$ for k_1, where $y_1 \epsilon N$in eq. (11) and using it again

$h(a, k_2, \ldots, k_m)\beta\lambda(k_1)\gamma[\lambda(y_1), \lambda(a)]_\gamma = 0$, for every $k_1, y_1, k_2, \ldots, k_m \epsilon N$,$\gamma \epsilon \mu$.

Since λ is an aut. , we get$h(a, k_2, \ldots, k_m)\ \mu\ N\ \mu[\lambda(y_1), \lambda(a)]_\gamma = \{0\}$.

Since λ is an aut. ,G is $\mu(N)$yields either$h(a, k_2, \ldots, k_m) = 0$ for every$k_2, \ldots, k_m \epsilon N$ or $a \epsilon Z(N)$.The proof is now complete.

References

[1]Asci. M., "*Γ-(σ, τ) Derivation on Gamma Near Rings*", InternationalMathematical Forum, Vol. 2, No. 3, pp. 97-102, 2007.[2] Ashraf, M. and Siddeeque, M. A., "*On (σ, τ)-n-derivations in near-rings",* Asian-European Journal of Mathematics, Vol.6, No. 4, 2013.

[3] Asraf, M. and Siddeeque, M. A., "*On permuting n-derivations in near-rings",*Commun. Kor. Math. Soc., Vol. 28, No. 4, pp. 697–707, 2013.

[4] Ashraf, M., Ali., A., and Ali, S., "*(σ, τ)-Derivations of prime near-rings", Arch.Math.* (*BRNO*),Vol. 40, pp. 281–286, 2004.

[5] L. Madhuchelvi, On Generalized Derivations in Γ-Near-Ring IntJr.of Mathematical sciences and Applications,ISSN N0, 6 (1) (2016) 2230-9888.

[6] Y. U. Cho. and Y. B. Jun. , Gamma-derivations in prime and semiprime gamma-near rings , Indian J. PureAppl. Math., 33(10) (2002) 1489–1494.

[7] R. Ravi, V.K. Bhat and k. Neetu , Commutativity of prime Γ-Near Rings with Γ-(σ, τ)-derivation, Math , 25(2009) 165-173.

[8] Shubham Sharma & Ahmed J. Obaid (2020) Mathematical modelling, analysis and design of fuzzy logic controller for the control of ventilation systems using MATLAB fuzzy logic toolbox, Journal of Interdisciplinary Mathematics, 23:4, 843-849, DOI: 10.1080/09720502.2020.1727611.

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 3708-3713
https://publishoa.com
ISSN: 1309-3452

[9] Obaid A.J. (2022) Data Mining Analysis Models Based on Prospective Detection of Infectious Disease. In: Sharma D.K., Peng SL., Sharma R., Zaitsev D.A. (eds) Micro-Electronics and Telecommunication Engineering. Lecture Notes in Networks and Systems, vol 373. Springer, Singapore. https://doi.org/10.1007/978-981-16-8721-1_41

[10] Azarshab M, Ghazanfari M, Heidarpoor F. An Intelligent Fuzzy Logic Based Traffic Controller. sjfst. 2021; 3 (1) :10-17
URL: http://sjfst.srpub.org/article-6-96-fa.html