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Transformed Kappa Distribution: Properties and Applications

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Abstract: The article, the three parameter KappaDistribution is fixed in a Larger Family Found by introducing an additional Parameter. We generalize The Three Parameter KappaDistribution by means of the Quadratic rank transmutation PlanStudied by Shaw et al. [9] to ,develop a Transmuted Kappa proppality, Distribution(TKD)Fourparameter . function, mean residual life function and stochastic ordering have been Explained. Maximum likelihood Estimation ,Its raw moments and,CentralMoments have been Obtained. The moment based measures including coefficient of Variation, Skewness, Kurtosis and index of dispersion have been discussed. The Statistical properties,Counting hazard Degree has, been discussed for Estimating the parameters ,of The distribution. Finally, Applications of the Distribution have been ,explained with one Examples of Observed real lifetime.

Keywords: TransformedKappa Distribution, kappa distribution, moments, Statistical Properties, maximum likelihood Estimation, Codness of fit

1. Introduction

The Statistical Researcher faces many statistical difficulties during the ,development of analyzing the data as well as Estimating the parameters related to the distribution, and among those problems that the researcher, directs is the process of determining the, appropriate and appropriate ,distribution of the data singularity.The Researchers Specialized in the ground specialized in the Statistical field have functioned to progress, the Probability Distributions and move them to ,the phase of the Transformed probability Distributions,in order to obtain, the best representation of the data with the least errors, Particularly ,when the Researcher appearances the Problem of Choosing The

,Sample With an Equal Probability,which Made The Original Distribution Limited to the Modelling of Phenomena It is unserviceable and when it develops necessary, to suggest a Specific Modification to be Done finished the TransformedSystem in Order to Obtain,Vocabulary That Has The Same Appearance of Accidents,

now a Days Transmuted Distributions And Their Mathematical Properties are Widely Studied For Applied Sciences Experimental Data Sets. transmuted rayleigh distribution (merovci, 2013), Transmuted Inverse, Rayleigh Distribution (Ahmad et al., (2014)), Transmuted Generalized Inverse weibull distribution (khan and king, 2013), transmuted modified inverse weibull distribution (elbatal, 2013), transmuted log-logistic distribution (aryal, (2013)), transmuted modified weibull dsistribution &transmuted lomax distribution (ashour Aand eltehiwy, (2013)), transmuted frechet distribution (mahmoud &mandouh, (2013)), transmuted pareto distribution (merovci &puka,(2014)), transmuted generalized gamma distribution (lucena et al., (2015)), transmuted weibull lomax distribution (afify et al.,(2015)) are reported with their various structural properties including explicit expressions for the Moments, Quantiles, Entropies, Mean Deviations And Order Statistics. all the Above Transmuted Distributions are Derived By Using quadratic Rank transmutation map(qRTM) Studied By Shaw & Buckley (2007). Report reveals that some properties of these distributions along with their parameters Are Estimated By Using Maximum Likelihood And bayesian Methods. usefulness Of some of These New Distributions Are Also illustrated With Experimental Data Sets. transmuted gumbel distribution (TGD) Along With Several Mathematical Properties Has Studied By aryal and tsokos ((2009)) Using quadratic rank transmutation map(QRTM)and Reported That(TGD)Can be Used to Model Climate Data. Therefore an Attempt Has Been Made to Developed transmuted exponentiated gumbel distribution (TEGD) Using exponentiated Gumbel distribution (EGD) ,and The quadratic rank transmutation map(QRTM). the parameters of The (TEGD) are Estimated by The Method Of Maximum Likelihood and Applied to The Water Quality Parameter Data Sets For Study The Usefulness Of The Model.

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A Random Variable X is Said to Have a Transmuted Distribution if its Cumulative Distribution Function (CDF) is Given By:

$G(x)=(1+\gamma)F(x)-\gamma[F(x)]^2 \qquad -1<\gamma<1 \quad (1)$

Where G(x) is the CDF of The Transmuted Distribution andF(x) is the CDFOF The base Distribution. differentiating (1) it Gives The Probability Density Function (PDF) ofThe Transmuted Distribution as:

$g(x) = f(x)[1+\gamma-2\gamma F(x)]$(2)

. The probability density function of Kappa Distributionis given by**:**

$$f(x,\alpha,\theta,\beta) = \frac{\alpha\theta}{\beta}\left(\frac{x}{\beta}\right)^{\theta-1}\left[\alpha+\left(\frac{x}{\beta}\right)^{\theta\alpha}\right]^{-\left(\frac{\alpha+1}{\alpha}\right)} ; x > 0, \theta\alpha,\theta,\beta > 0 \quad (3)$$

and the cumulative distribution function of Kappa Distribution(KD) is given by:

$$F(x,\theta) = \left[\frac{\left(\frac{x}{\beta}\right)^{\theta\alpha}}{\alpha+\left(\frac{x}{\beta}\right)^{\theta\alpha}}\right]^{\left(\frac{1}{\alpha}\right)} ; x > 0, \alpha,\theta,\beta > 0 \qquad (4)$$

Using (1) The CDF of Transformed Kappa Distribution (TKD) for:

$$F(x,\alpha,\theta,\beta,\gamma) = \left(\alpha+\left(\frac{x}{\beta}\right)^{\theta\alpha}\right)^{\frac{-1}{\alpha}}\left(\frac{x}{\beta}\right)(1+\gamma)-\gamma\left(\alpha+\left(\frac{x}{\beta}\right)^{\alpha\theta}\right)^{-2/\alpha}\left(\frac{x}{\beta}\right)^{2\theta} \quad (5)$$

Where: α,θ,β is Scale parameter.γ : is shape and scale Parameters

the CDF plots of the TKD are displayed in fig. 1

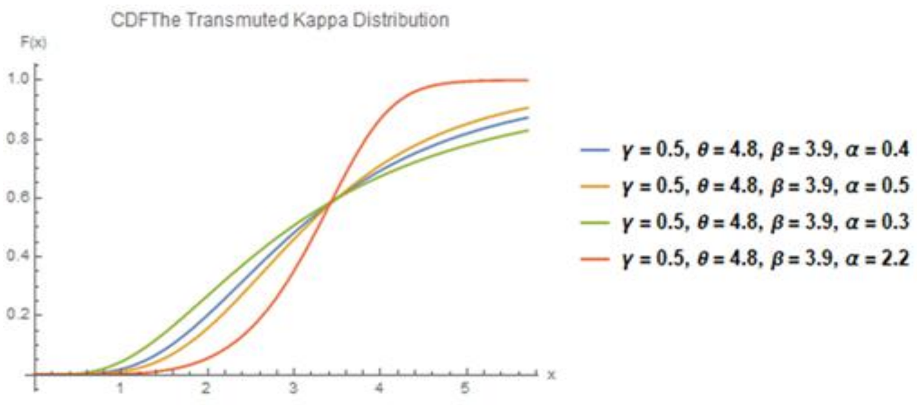

FIGURE 1. The Shape for the CDF of the TKD distribution

Using (2) The PDF of Transformed Kappa Distribution (TKD) for:

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$$f(x,\alpha,\theta,\beta,\gamma)=\frac{\alpha\theta}{\beta}\left(\frac{x}{\beta}\right)^{\theta-1}\left[\alpha+\left(\frac{x}{\beta}\right)^{\theta\alpha}\right]^{-\left(\frac{\alpha+1}{\alpha}\right)}\left[1+\gamma-2\gamma\left[\frac{\left(\frac{x}{\beta}\right)^{\alpha\theta}}{\alpha+\left(\frac{x}{\beta}\right)^{\alpha\theta}}\right]^{\frac{1}{\alpha}}\right] \quad |\gamma|<1\ ;x,\alpha,\theta,\beta,\gamma\geq 0 \quad (6)$$

the PDF plots of the TKD are displayed in fig. 2

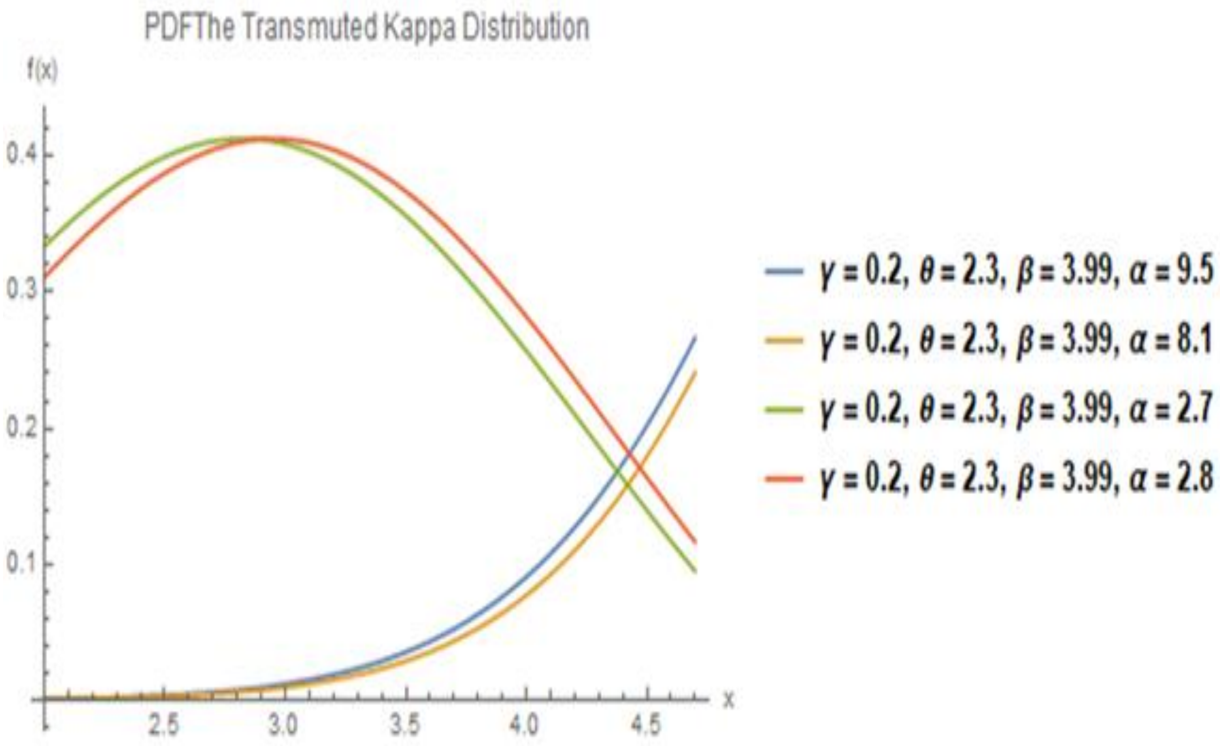

FIGURE 2. The Shape for the PDF of the TKD distribution

the reliability Function or the Survival Function O of Transformed Kappa Distribution (TKD) is calculated as:

$S(x) = 1 - F(x,\alpha,\theta,\beta,\gamma)$

$$\mathbf{S}(x,\alpha,\theta,\beta,\gamma) = \mathbf{1} - \left(\alpha+\left(\frac{x}{\beta}\right)^{\theta\alpha}\right)^{\frac{-1}{\alpha}}\left(\frac{x}{\beta}\right)(1+\gamma) - \gamma\left(\alpha+\left(\frac{x}{\beta}\right)^{\alpha\theta}\right)^{-2/\alpha}\left(\frac{x}{\beta}\right)^{2\theta} \quad (7)$$

the Survival Function plots of the (TKD) are displayed in fig. 3

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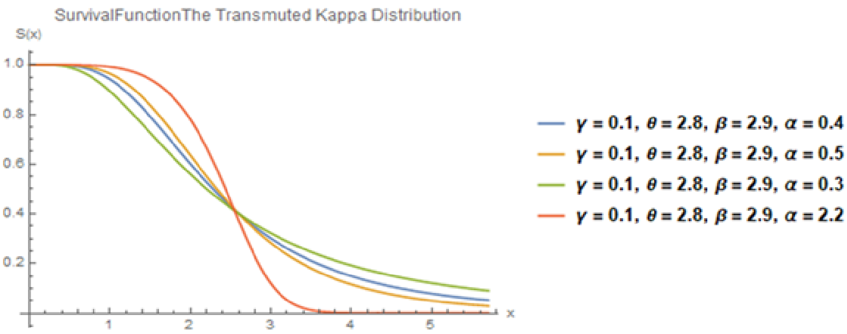


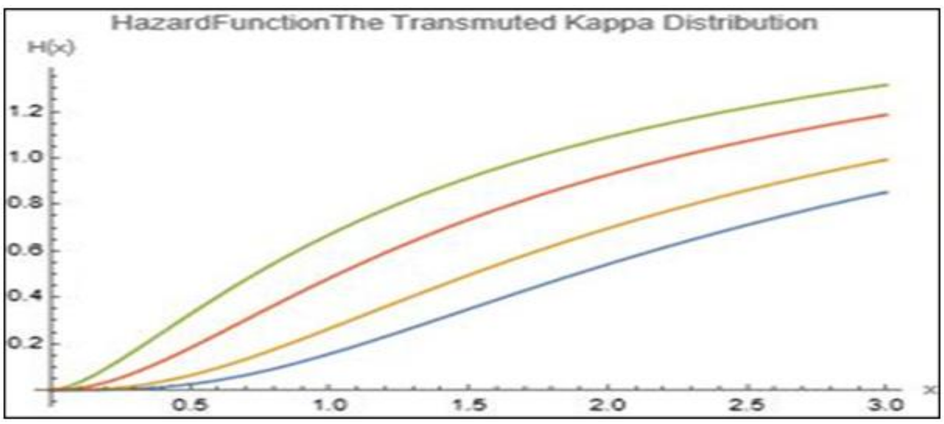
FIGURE 3. The Shape for the Survival Functionof the TKD distribution

The Hazard Function is Also known as Hazard rate and is Called as the Immediate failure rate or force of mortality and the hazard function of Transformed Kappa Distribution (TKD) is given by:

$$H(x,\alpha,\theta,\beta,\gamma) = \frac{f(x,\alpha,\theta,\beta,\gamma)}{S(x,\alpha,\theta,\beta,\gamma)}$$

$$H(x,\alpha,\theta,\beta,\gamma) = \frac{\frac{\alpha\theta}{\beta}\left(\frac{x}{\beta}\right)^{\theta-1}\left[\alpha+\left(\frac{x}{\beta}\right)^{\theta\alpha}\right]^{-\left(\frac{\alpha+1}{\alpha}\right)}\left[1+\gamma-2\gamma\left[\frac{\left(\frac{x}{\beta}\right)^{\alpha\theta}}{\alpha+\left(\frac{x}{\beta}\right)^{\alpha\theta}}\right]^{\frac{1}{\alpha}}\right]}{1-\left(\alpha+\left(\frac{x}{\beta}\right)^{\theta\alpha}\right)^{\frac{-1}{\alpha}}\left(\frac{x}{\beta}\right)(1+\gamma)-\gamma\left(\alpha+\left(\frac{x}{\beta}\right)^{\alpha\theta}\right)^{-2/\alpha}\left(\frac{x}{\beta}\right)^{2\theta}} \quad \mathbf{(8)}$$

The Survival Function plots hazard function of Transformed Kappa Distributionare displayed in fig. 4



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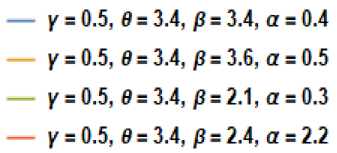

2. STATISTICAL MEASURES

in This Portion, we Have Obtained The Different Statistical Properties of **Transformed Kappa Distribution**Distribution. a moments

Let X Denotes the random Variable Of transformed Kappa distribution the r th order moment$EX^r$of Transformed Kappa n Distribution about origin is:

$$\mathbf{EX^r} = \int_{\mathbf{0}}^{\infty} \mathbf{x^r}\, \mathbf{f}(x,\alpha,\theta,\beta,\gamma) \quad \mathbf{dx}$$

$$\mathbf{EX^r} = \int_{\mathbf{0}}^{\infty} \mathbf{x^r} \frac{\alpha\theta}{\beta} \left(\frac{x}{\beta}\right)^{\theta-1} \left[\alpha+\left(\frac{x}{\beta}\right)^{\theta\alpha}\right]^{-\left(\frac{\alpha+1}{\alpha}\right)} \left[1+\gamma-2\gamma\left[\frac{\left(\frac{x}{\beta}\right)^{\alpha\theta}}{\alpha+\left(\frac{x}{\beta}\right)^{\alpha\theta}}\right]^{\frac{1}{\alpha}}\right] \quad \mathbf{dx}$$

$$EX^r = \beta^r \alpha^{\frac{r}{\alpha\theta}-1}\left[\frac{\Gamma\frac{r+\theta}{\alpha\theta}\Gamma 1-\frac{r}{\alpha\theta}}{\Gamma\frac{\alpha+1}{\alpha}}\right] + \beta^r \alpha^{\frac{r}{\alpha\theta}-1}\left[\frac{\Gamma 1-\frac{r}{\alpha\theta}\Gamma\frac{r+2\theta}{\alpha\theta}}{\Gamma\frac{\alpha+2}{\alpha}}\right]$$

Where r=1

$$Ex^1=\mu'_1=\beta^1\alpha^{\frac{1}{\alpha\theta}-1}\left[\frac{\Gamma\frac{1+\theta}{\alpha\theta}\Gamma 1-\frac{1}{\alpha\theta}}{\Gamma\frac{\alpha+1}{\alpha}}\right](1+\gamma)-2\gamma\beta^1\alpha^{\frac{1}{\alpha\theta}-1}\left[\frac{\Gamma 1-\frac{1}{\alpha\theta}\Gamma\frac{1+2\theta}{\alpha\theta}}{\Gamma\frac{\alpha+2}{\alpha}}\right] \quad (\mathbf{9})$$

Where r=2

$$Ex^2=\mu'_2=\beta^2\alpha^{\frac{2}{\alpha\theta}-1}\left[\frac{\Gamma\frac{2+\theta}{\alpha\theta}\Gamma 1-\frac{2}{\alpha\theta}}{\Gamma\frac{\alpha+1}{\alpha}}\right](1+\gamma)-2\gamma\beta^2\alpha^{\frac{2}{\alpha\theta}-1}\left[\frac{\Gamma 1-\frac{2}{\alpha\theta}\Gamma\frac{2+2\theta}{\alpha\theta}}{\Gamma\frac{\alpha+2}{\alpha}}\right] \quad (\mathbf{10})$$

Where r=3

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$$Ex^3=\mu'_3=\beta^3\alpha^{\frac{3}{\alpha\theta}-1}\left[\frac{\Gamma\frac{3+\theta}{\alpha\theta}\Gamma 1-\frac{3}{\alpha\theta}}{\Gamma\frac{\alpha+1}{\alpha}}\right](1+\gamma)-2\gamma\beta^3\alpha^{\frac{3}{\alpha\theta}-1}\left[\frac{\Gamma 1-\frac{3}{\alpha\theta}\Gamma\frac{3+2\theta}{\alpha\theta}}{\Gamma\frac{\alpha+2}{\alpha}}\right] \quad (\mathbf{11})$$

Where r=4

$$Ex^4=\mu'_4=\beta^4\alpha^{\frac{4}{\alpha\theta}-1}\left[\frac{\Gamma\frac{4+\theta}{\alpha\theta}\Gamma 1-\frac{4}{\alpha\theta}}{\Gamma\frac{\alpha+1}{\alpha}}\right](1+\gamma)-2\gamma\beta^4\alpha^{\frac{4}{\alpha\theta}-1}\left[\frac{\Gamma 1-\frac{4}{\alpha\theta}\Gamma\frac{4+2\theta}{\alpha\theta}}{\Gamma\frac{\alpha+2}{\alpha}}\right] \quad (\mathbf{12})$$

using Relationship Between Central Moments (Moments About The Mean) and Moments About Origin, The Central Moments of Transformed Kappa n Distribution About Origin is:

$$E(x-\mu)^r=\int_0^\infty (x-\mu)^r\,\mathbf{f}\,(x,\alpha,\theta,\beta,\gamma).\,dx$$

$$E(x-\mu)^r=\int_0^\infty (x-\mu)^r\frac{\alpha\theta}{\beta}\left(\frac{x}{\beta}\right)^{\theta-1}\left[\alpha+\left(\frac{x}{\beta}\right)^{\theta\alpha}\right]^{-\left(\frac{\alpha+1}{\alpha}\right)}\left[1+\gamma-2\gamma\left[\frac{\left(\frac{x}{\beta}\right)^{\alpha\theta}}{\alpha+\left(\frac{x}{\beta}\right)^{\alpha\theta}}\right]^{\frac{1}{\alpha}}\right].\,dx$$

$$(x-\mu)^r=\begin{pmatrix}\beta^j\alpha^{\frac{j}{\theta\alpha}-1}\sum_{j=0}^{r}\binom{r}{j}(-\mu)^{r-j}\beta^j\alpha^{\frac{j}{\theta\alpha}-1}\left[\frac{\Gamma\frac{j}{\alpha\theta}+\frac{1}{\alpha}\Gamma 1-\frac{j}{\alpha\theta}}{\Gamma 1+\frac{1}{\alpha}}\right](1+\gamma)\\ -2\gamma\beta^r\alpha^{\frac{r}{\theta\alpha}-1}\sum_{j=0}^{r}\binom{r}{j}\left(-\frac{\mu}{\alpha^{\frac{1}{\theta\alpha}}\beta}\right)^{r-j}\left[\frac{\Gamma\frac{j}{\alpha\theta}+\frac{2}{\alpha}\Gamma 1-\frac{j}{\alpha\theta}-\frac{1}{\alpha}}{\Gamma 1+\frac{1}{\alpha}}\right]\end{pmatrix}$$

Where r=2

$$(x-\mu)^2=\begin{pmatrix}\beta^j\alpha^{\frac{j}{\theta\alpha}-1}\sum_{j=0}^{2}\binom{2}{j}(-\mu)^{2-j}\beta^j\alpha^{\frac{j}{\theta\alpha}-1}\left[\frac{\Gamma\frac{j}{\alpha\theta}+\frac{1}{\alpha}\Gamma 1-\frac{j}{\alpha\theta}}{\Gamma 1+\frac{1}{\alpha}}\right](1+\gamma))\\ -2\gamma\beta^2\alpha^{\frac{2}{\theta\alpha}-1}\sum_{j=0}^{2}\binom{2}{j}\left(-\frac{\mu}{\alpha^{\frac{1}{\theta\alpha}}\beta}\right)^{2-j}\left[\frac{\Gamma\frac{j}{\alpha\theta}+\frac{2}{\alpha}\Gamma 1-\frac{j}{\alpha\theta}-\frac{1}{\alpha}}{\Gamma 1+\frac{1}{\alpha}}\right]\end{pmatrix} \quad (\mathbf{13})$$

$$\sigma^2=\begin{pmatrix}\beta^j\alpha^{\frac{j}{\theta\alpha}-1}\sum_{j=0}^{2}\binom{2}{j}(-\mu)^{2-j}\beta^j\alpha^{\frac{j}{\theta\alpha}-1}\left[\frac{\Gamma\frac{j}{\alpha\theta}+\frac{1}{\alpha}\Gamma 1-\frac{j}{\alpha\theta}}{\Gamma 1+\frac{1}{\alpha}}\right](1+\gamma))\\ -2\gamma\beta^2\alpha^{\frac{2}{\theta\alpha}-1}\sum_{j=0}^{2}\binom{2}{j}\left(-\frac{\mu}{\alpha^{\frac{1}{\theta\alpha}}\beta}\right)^{2-j}\left[\frac{\Gamma\frac{j}{\alpha\theta}+\frac{2}{\alpha}\Gamma 1-\frac{j}{\alpha\theta}-\frac{1}{\alpha}}{\Gamma 1+\frac{1}{\alpha}}\right]\end{pmatrix}$$

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$$\sigma = \sqrt{\begin{pmatrix} \beta^{j}\alpha^{\frac{j}{\theta\alpha}-1}\sum_{j=0}^{2}\binom{2}{j}(-\mu)^{2-j}\beta^{j}\alpha^{\frac{j}{\theta\alpha}-1}\left[\frac{\Gamma\frac{j}{\alpha\theta}+\frac{1}{\alpha}\Gamma 1-\frac{j}{\alpha\theta}}{\Gamma 1+\frac{1}{\alpha}}\right](1+\gamma)) \\ -2\gamma\beta^{2}\alpha^{\frac{2}{\theta\alpha}-1}\sum_{j=0}^{2}\binom{2}{j}\left(-\frac{\mu}{\alpha^{\frac{1}{\theta\alpha}}\beta}\right)^{2-j}\left[\frac{\Gamma\frac{j}{\alpha\theta}+\frac{2}{\alpha}\Gamma 1-\frac{j}{\alpha\theta}-\frac{1}{\alpha}}{\Gamma 1+\frac{1}{\alpha}}\right] \end{pmatrix}}$$

Coefficients of Variation (C.V)

$$CV = \frac{\sigma}{\mu} \times 100$$

$$C.V = \frac{\sqrt{\begin{matrix} \beta^{j}\alpha^{\frac{j}{\theta\alpha}-1}\sum_{j=0}^{2}\binom{2}{j}(-\mu)^{2-j}\beta^{j}\alpha^{\frac{j}{\theta\alpha}-1}\left[\frac{\Gamma\frac{j}{\alpha\theta}+\frac{1}{\alpha}\Gamma 1-\frac{j}{\alpha\theta}}{\Gamma 1+\frac{1}{\alpha}}\right](1+\gamma) \\ -2\gamma\beta^{2}\alpha^{\frac{2}{\theta\alpha}-1}\sum_{j=0}^{2}\binom{2}{j}\left(-\frac{\mu}{\alpha^{\frac{1}{\theta\alpha}}\beta}\right)^{2-j}\left[\frac{\Gamma\frac{j}{\alpha\theta}+\frac{2}{\alpha}\Gamma 1-\frac{j}{\alpha\theta}-\frac{1}{\alpha}}{\Gamma 1+\frac{1}{\alpha}}\right] \end{matrix}}}{\beta^{1}\alpha^{\frac{1}{\alpha\theta}-1}\left[\frac{\Gamma\frac{1+\theta}{\alpha\theta}\Gamma 1-\frac{1}{\alpha\theta}}{\Gamma\frac{\alpha+1}{\alpha}}\right](1+\gamma)-2\gamma\beta^{1}\alpha^{\frac{1}{\alpha\theta}-1}\left[\frac{\Gamma 1-\frac{1}{\alpha\theta}\Gamma\frac{1+2\theta}{\alpha\theta}}{\Gamma\frac{\alpha+2}{\alpha}}\right]} \times 100 \quad (\mathbf{14})$$

Where r=3

$$(x-\mu)^{3}=\begin{pmatrix} \beta^{j}\alpha^{\frac{j}{\theta\alpha}-1}\sum_{j=0}^{3}\binom{3}{j}(-\mu)^{3-j}\beta^{j}\alpha^{\frac{j}{\theta\alpha}-1}\left[\frac{\Gamma\frac{j}{\alpha\theta}+\frac{1}{\alpha}\Gamma 1-\frac{j}{\alpha\theta}}{\Gamma 1+\frac{1}{\alpha}}\right](1+\gamma) \\ -2\gamma\beta^{3}\alpha^{\frac{3}{\theta\alpha}-1}\sum_{j=0}^{3}\binom{3}{j}\left(-\frac{\mu}{\alpha^{\frac{1}{\theta\alpha}}\beta}\right)^{3-j}\left[\frac{\Gamma\frac{j}{\alpha\theta}+\frac{2}{\alpha}\Gamma 1-\frac{j}{\alpha\theta}-\frac{1}{\alpha}}{\Gamma 1+\frac{1}{\alpha}}\right] \end{pmatrix}$$

Where r=4

$$(x-\mu)^{4}=\begin{pmatrix} \beta^{j}\alpha^{\frac{j}{\theta\alpha}-1}\sum_{j=0}^{4}\binom{4}{j}(-\mu)^{4-j}\beta^{j}\alpha^{\frac{j}{\theta\alpha}-1}\left[\frac{\Gamma\frac{j}{\alpha\theta}+\frac{1}{\alpha}\Gamma 1-\frac{j}{\alpha\theta}}{\Gamma 1+\frac{1}{\alpha}}\right](1+\gamma) \\ -2\gamma\beta^{4}\alpha^{\frac{4}{\theta\alpha}-1}\sum_{j=0}^{4}\binom{4}{j}\left(-\frac{\mu}{\alpha^{\frac{1}{\theta\alpha}}\beta}\right)^{4-j}\left[\frac{\Gamma\frac{j}{\alpha\theta}+\frac{2}{\alpha}\Gamma 1-\frac{j}{\alpha\theta}-\frac{1}{\alpha}}{\Gamma 1+\frac{1}{\alpha}}\right] \end{pmatrix}$$

Coefficient of Skewness

$$S.K = \frac{\mu_3}{(\mu_2)^{\frac{3}{2}}}$$

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$$\text{S.K}=\frac{\begin{bmatrix}\beta^j\alpha^{\frac{j}{\theta\alpha}-1}\sum_{j=0}^{3}\binom{3}{j}(-\mu)^{3-j}\beta^j\alpha^{\frac{j}{\theta\alpha}-1}\left[\frac{\Gamma\frac{j}{\alpha\theta}+\frac{1}{\alpha}\Gamma 1-\frac{j}{\alpha\theta}}{\Gamma 1+\frac{1}{\alpha}}\right](1+\gamma)\\ -2\gamma\beta^3\alpha^{\frac{3}{\theta\alpha}-1}\sum_{j=0}^{3}\binom{3}{j}\left(-\frac{\mu}{\alpha^{\frac{1}{\theta\alpha}}\beta}\right)^{3-j}\left[\frac{\Gamma\frac{j}{\alpha\theta}+\frac{2}{\alpha}\Gamma 1-\frac{j}{\alpha\theta}-\frac{1}{\alpha}}{\Gamma 1+\frac{1}{\alpha}}\right]\end{bmatrix}}{\begin{bmatrix}\beta^j\alpha^{\frac{j}{\theta\alpha}-1}\sum_{j=0}^{2}\binom{2}{j}(-\mu)^{2-j}\beta^j\alpha^{\frac{j}{\theta\alpha}-1}\left[\frac{\Gamma\frac{j}{\alpha\theta}+\frac{1}{\alpha}\Gamma 1-\frac{j}{\alpha\theta}}{\Gamma 1+\frac{1}{\alpha}}\right](1+\gamma)\\ -2\gamma\beta^2\alpha^{\frac{2}{\theta\alpha}-1}\sum_{j=0}^{2}\binom{2}{j}\left(-\frac{\mu}{\alpha^{\frac{1}{\theta\alpha}}\beta}\right)^{2-j}\left[\frac{\Gamma\frac{j}{\alpha\theta}+\frac{2}{\alpha}\Gamma 1-\frac{j}{\alpha\theta}-\frac{1}{\alpha}}{\Gamma 1+\frac{1}{\alpha}}\right]\end{bmatrix}^{\frac{3}{2}}}\,(\mathbf{15})$$

Coefficient of Kurtosis

$$\text{C.K}=\left(\frac{(x-\mu)^4}{\sigma^4}\right)$$

$$\text{C.K}=\frac{(x-\mu)^4}{((x-\mu)^2)^2}$$

$$\text{C.K}=\left[\frac{\begin{pmatrix}\beta^j\alpha^{\frac{j}{\theta\alpha}-1}\sum_{j=0}^{4}\binom{4}{j}(-\mu)^{4-j}\beta^j\alpha^{\frac{j}{\theta\alpha}-1}\left[\frac{\Gamma\frac{j}{\alpha\theta}+\frac{1}{\alpha}\Gamma 1-\frac{j}{\alpha\theta}}{\Gamma 1+\frac{1}{\alpha}}\right](1+\gamma)\\ -2\gamma\beta^4\alpha^{\frac{4}{\theta\alpha}-1}\sum_{j=0}^{4}\binom{4}{j}\left(-\frac{\mu}{\alpha^{\frac{1}{\theta\alpha}}\beta}\right)^{4-j}\left[\frac{\Gamma\frac{j}{\alpha\theta}+\frac{2}{\alpha}\Gamma 1-\frac{j}{\alpha\theta}-\frac{1}{\alpha}}{\Gamma 1+\frac{1}{\alpha}}\right]\end{pmatrix}}{\begin{pmatrix}\beta^j\alpha^{\frac{j}{\theta\alpha}-1}\sum_{j=0}^{2}\binom{2}{j}(-\mu)^{2-j}\beta^j\alpha^{\frac{j}{\theta\alpha}-1}\left[\frac{\Gamma\frac{j}{\alpha\theta}+\frac{1}{\alpha}\Gamma 1-\frac{j}{\alpha\theta}}{\Gamma 1+\frac{1}{\alpha}}\right](1+\gamma)\\ -2\gamma\beta^2\alpha^{\frac{2}{\theta\alpha}-1}\sum_{j=0}^{2}\binom{2}{j}\left(-\frac{\mu}{\alpha^{\frac{1}{\theta\alpha}}\beta}\right)^{2-j}\left[\frac{\Gamma\frac{j}{\alpha\theta}+\frac{2}{\alpha}\Gamma 1-\frac{j}{\alpha\theta}-\frac{1}{\alpha}}{\Gamma 1+\frac{1}{\alpha}}\right]\end{pmatrix}^{2}}\right](\mathbf{16})$$

3.parameter estimation

the Method of Maximum Likelihood Estimate is Used for Estimating The Parameters of The Newly Proposed Distribution Known as OF The Transformed Kappa Distribution. Let x1, x2,…,xn be a Random Sample of Ssize n From OFThe Transformed Kappa Distribution, Fhen the Corresponding likelihood Function is Given By:

$$\text{Lf}(x_1.x_2.\dots.x_n)=\prod_{i=1}^{n}\mathbf{f}(x,\alpha,\theta,\beta,\gamma)\ \dots\ (11)$$

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$$\mathrm{Lf}(xi,\beta,\alpha,\theta,\gamma)=\prod_{i=1}^{n}\left[\frac{\alpha\theta}{\beta}\left(\frac{x}{\beta}\right)^{\theta-1}\left[\alpha+\left(\frac{x}{\beta}\right)^{\theta\alpha}\right]^{-\left(\frac{\alpha+1}{\alpha}\right)}\left[1+-2\gamma\left[\frac{\left(\frac{x}{\beta}\right)^{\alpha\theta}}{\alpha+\left(\frac{x}{\beta}\right)^{\alpha\theta}}\right]^{\frac{1}{\alpha}}\right]\right]\quad(\mathbf{17})$$

$$=\prod_{i=1}^{n}\left[\frac{\alpha\theta}{\beta}\left(\frac{x}{\beta}\right)^{\theta-1}\left[\alpha+\left(\frac{x}{\beta}\right)^{\theta\alpha}\right]^{-\left(\frac{\alpha+1}{\alpha}\right)}\left[1+-2\gamma\left[\frac{\left(\frac{x}{\beta}\right)^{\alpha\theta}}{\alpha+\left(\frac{x}{\beta}\right)^{\alpha\theta}}\right]^{\frac{1}{\alpha}}\right]\right]$$

$$=\frac{\alpha^{n}\theta^{n}}{\beta^{n}}\prod_{i=1}^{n}\left[\left(\frac{x}{\beta}\right)^{\theta-1}\left[\alpha+\left(\frac{x}{\beta}\right)^{\theta\alpha}\right]^{-\left(\frac{\alpha+1}{\alpha}\right)}\left[1+\gamma-2\gamma\left[\frac{\left(\frac{x}{\beta}\right)^{\alpha\theta}}{\alpha+\left(\frac{x}{\beta}\right)^{\alpha\theta}}\right]^{\frac{1}{\alpha}}\right]\right]$$

$$\mathrm{logLf}(xi,\beta,\alpha,\theta,\gamma)=\left\{\begin{array}{c} n\mathrm{Ln}\alpha+n\mathrm{Ln}\theta-n\mathrm{Ln}\beta+(\theta-1)\sum_{i=1}^{n}\mathrm{Ln}\left(\frac{xi}{\beta}\right)-\left(\frac{\alpha+1}{\alpha}\right)\sum_{i=1}^{n}\mathrm{Ln}\left(\alpha+\left(\frac{xi}{\beta}\right)^{\alpha\theta}\right)\\ +\sum_{i=1}^{n}\mathrm{Ln}\left[1+\gamma-2\gamma\left[\frac{\left(\frac{x}{\beta}\right)^{\alpha\theta}}{\alpha+\left(\frac{x}{\beta}\right)^{\alpha\theta}}\right]^{\frac{1}{\alpha}}\right]\end{array}\right\}$$

$$\frac{\mathrm{logLf}(xi,\beta,\alpha,\theta,\gamma)}{d\beta}=\left\{\begin{array}{c}-\frac{n}{\beta}-\frac{n(-1+\theta)}{\beta}+\frac{nx(1+\alpha)\left(\frac{x}{\beta}\right)^{-1+\alpha\theta}\theta}{\left(\alpha+\left(\frac{x}{\beta}\right)^{\alpha\theta}\right)\beta^{2}}-\\ \frac{2n\left(\frac{\left(\frac{x}{\beta}\right)^{\alpha\theta}}{\alpha+\left(\frac{x}{\beta}\right)^{\alpha\theta}}\right)^{-1+\frac{1}{\alpha}}\left(-\frac{x\alpha\left(\frac{x}{\beta}\right)^{-1+\alpha\theta}\theta}{\left(\alpha+\left(\frac{x}{\beta}\right)^{\alpha\theta}\right)\beta^{2}}+\frac{x\alpha\left(\frac{x}{\beta}\right)^{-1+2\alpha\theta}\theta}{\left(\alpha+\left(\frac{x}{\beta}\right)^{\alpha\theta}\right)^{2}\beta^{2}}\right)\gamma}{\alpha\left(1+\gamma-2\left(\frac{\left(\frac{x}{\beta}\right)^{\alpha\theta}}{\alpha+\left(\frac{x}{\beta}\right)^{\alpha\theta}}\right)^{\frac{1}{\alpha}}\gamma\right)}\end{array}\right\}=0\quad(\mathbf{18})$$

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$$\frac{\log Lf(xi,\beta,\alpha,\theta,\gamma)}{d\theta}=\left\{\begin{array}{c}\frac{n}{\theta}+n\mathrm{Log}\left[\frac{x}{\beta}\right]-\frac{n(1+\alpha)\left(\frac{x}{\beta}\right)^{\alpha\theta}\mathrm{Log}\left[\frac{x}{\beta}\right]}{\alpha+\left(\frac{x}{\beta}\right)^{\alpha\theta}}\\-\frac{2n\left(\frac{\left(\frac{x}{\beta}\right)^{\alpha\theta}}{\alpha+\left(\frac{x}{\beta}\right)^{\alpha\theta}}\right)^{-1+\frac{1}{\alpha}}\gamma\left(\frac{\alpha\left(\frac{x}{\beta}\right)^{\alpha\theta}\mathrm{Log}\left[\frac{x}{\beta}\right]}{\alpha+\left(\frac{x}{\beta}\right)^{\alpha\theta}}-\frac{\alpha\left(\frac{x}{\beta}\right)^{2\alpha\theta}\mathrm{Log}\left[\frac{x}{\beta}\right]}{\left(\alpha+\left(\frac{x}{\beta}\right)^{\alpha\theta}\right)^{2}}\right)}{\alpha\left(1+\gamma-2\left(\frac{\left(\frac{x}{\beta}\right)^{\alpha\theta}}{\alpha+\left(\frac{x}{\beta}\right)^{\alpha\theta}}\right)^{\frac{1}{\alpha}}\gamma\right)}\end{array}\right\}=0(\mathbf{19})$$

$$\frac{\log Lf(xi,\beta,\alpha,\theta,\gamma)}{d\boldsymbol{\alpha}}$$

$$=\left\{\begin{array}{c}=\frac{\mathbf{n}}{\boldsymbol{\alpha}}-\frac{\mathbf{nLog}\left[\boldsymbol{\alpha}+\left(\frac{\mathbf{x}}{\boldsymbol{\beta}}\right)^{\boldsymbol{\alpha\theta}}\right]}{\boldsymbol{\alpha}}+\frac{\mathbf{n(1+\alpha)Log}\left[\boldsymbol{\alpha}+\left(\frac{\mathbf{x}}{\boldsymbol{\beta}}\right)^{\boldsymbol{\alpha\theta}}\right]}{\boldsymbol{\alpha}^2}-\frac{\mathbf{n(1+\alpha)}\left(\mathbf{1}+\left(\frac{\mathbf{x}}{\boldsymbol{\beta}}\right)^{\boldsymbol{\alpha\theta}}\boldsymbol{\theta}\mathbf{Log}\left[\frac{\mathbf{x}}{\boldsymbol{\beta}}\right]\right)}{\boldsymbol{\alpha}\left(\boldsymbol{\alpha}+\left(\frac{\mathbf{x}}{\boldsymbol{\beta}}\right)^{\boldsymbol{\alpha\theta}}\right)}-\\ \frac{\mathbf{2n}\left(\frac{\left(\frac{\mathbf{x}}{\boldsymbol{\beta}}\right)^{\boldsymbol{\alpha\theta}}}{\boldsymbol{\alpha}+\left(\frac{\mathbf{x}}{\boldsymbol{\beta}}\right)^{\boldsymbol{\alpha\theta}}}\right)^{\frac{1}{\boldsymbol{\alpha}}}\boldsymbol{\gamma}\left(-\frac{\mathbf{Log}\left[\frac{\left(\frac{\mathbf{x}}{\boldsymbol{\beta}}\right)^{\boldsymbol{\alpha\theta}}}{\boldsymbol{\alpha}+\left(\frac{\mathbf{x}}{\boldsymbol{\beta}}\right)^{\boldsymbol{\alpha\theta}}}\right]}{\boldsymbol{\alpha}^2}\right)}{\mathbf{1}+\boldsymbol{\gamma}-\mathbf{2}\left(\frac{\left(\frac{\mathbf{x}}{\boldsymbol{\beta}}\right)^{\boldsymbol{\alpha\theta}}}{\boldsymbol{\alpha}+\left(\frac{\mathbf{x}}{\boldsymbol{\beta}}\right)^{\boldsymbol{\alpha\theta}}}\right)^{\frac{1}{\boldsymbol{\alpha}}}\boldsymbol{\gamma}}\\ \frac{\frac{\left(\boldsymbol{\alpha}+\left(\frac{\mathbf{x}}{\boldsymbol{\beta}}\right)^{\boldsymbol{\alpha\theta}}\right)\left(\frac{\mathbf{x}}{\boldsymbol{\beta}}\right)^{-\boldsymbol{\alpha\theta}}\left(\frac{\left(\frac{\mathbf{x}}{\boldsymbol{\beta}}\right)^{\boldsymbol{\alpha\theta}}\boldsymbol{\theta}\mathbf{Log}\left[\frac{\mathbf{x}}{\boldsymbol{\beta}}\right]}{\boldsymbol{\alpha}+\left(\frac{\mathbf{x}}{\boldsymbol{\beta}}\right)^{\boldsymbol{\alpha\theta}}}-\frac{\left(\frac{\mathbf{x}}{\boldsymbol{\beta}}\right)^{\boldsymbol{\alpha\theta}}\left(\mathbf{1}+\left(\frac{\mathbf{x}}{\boldsymbol{\beta}}\right)^{\boldsymbol{\alpha\theta}}\boldsymbol{\theta}\mathbf{Log}\left[\frac{\mathbf{x}}{\boldsymbol{\beta}}\right]\right)}{\left(\boldsymbol{\alpha}+\left(\frac{\mathbf{x}}{\boldsymbol{\beta}}\right)^{\boldsymbol{\alpha\theta}}\right)^{2}}\right)}{\boldsymbol{\alpha}}}{\mathbf{1}+\boldsymbol{\gamma}-\mathbf{2}\left(\frac{\left(\frac{\mathbf{x}}{\boldsymbol{\beta}}\right)^{\boldsymbol{\alpha\theta}}}{\boldsymbol{\alpha}+\left(\frac{\mathbf{x}}{\boldsymbol{\beta}}\right)^{\boldsymbol{\alpha\theta}}}\right)^{\frac{1}{\boldsymbol{\alpha}}}\boldsymbol{\gamma}}\end{array}\right\}$$

$$=0(\mathbf{20})$$

$$\frac{\log Lf(xi,\beta,\alpha,\theta,\gamma)}{d\boldsymbol{\gamma}}=\left\{\frac{n\left(1-2\left(\frac{\left(\frac{x}{\beta}\right)^{\alpha\theta}}{\alpha+\left(\frac{x}{\beta}\right)^{\alpha\theta}}\right)^{\frac{1}{\alpha}}\right)}{1+\gamma-2\left(\frac{\left(\frac{x}{\beta}\right)^{\alpha\theta}}{\alpha+\left(\frac{x}{\beta}\right)^{\alpha\theta}}\right)^{\frac{1}{\alpha}}\gamma}\right\}=0(\mathbf{21})$$

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Equations (18), (19),), (20) and (21) Cannot be Solved By The Usual Analytical Methods Because They are Non-Linear Equations and Therefore They Were Solved using the numerical method (nelder-mead) to obtain the Estimations of the Greatest Possibility Method.

4.APPLICATION OF The Transformed Kappa Distribution (TKD)

the Flexibility and performance of The Transformed Kappa Distributionare Evaluated on Competing models viz One parameter Exponential Distribution (ED), Three Parameter lindely Distribution(TPLD), Gamma Distribution(G.D), and Three Parameter kappa Distribution (KD). and Weibull Distribution(WD). Here, the distribution is fitted to data set for the number ofweeksThe Patients people withheart diseasewere in hospital before death For AL hussein Educational Hospital in Karbala, for sample size (n=104) (see table 1.), the performance of the distribution was compared with exponential, Exponential Pareto, Lindely Three parametric, weibel distribution and weibel Paretodistribution for the data set using akaike information Criterion (AIC),(BIC), Akaike Information Criterion Corrected (AICC). Distribution with the lowest AIC, AICC considered the most Flexible and Superior Distribution For a Given Data Set.The Results are Presented in The Tables (2).

TABLE 1. Data set for the number of weeks The Patients people with heart

0.1	0.3	1.2	1.4	1.6	2	2.6	3	3.5	4.1	4.8
0.1	0.3	1.2	1.4	1.7	2.1	2.7	3.1	3.6	4.3	4.8
0.1	0.4	1.2	1.5	1.7	2.2	2.7	3.1	3.6	4.4	4.9
0.2	0.4	1.3	1.5	1.7	2.4	2.8	3.1	3.6	4.4	4.9
0.2	0.4	1.3	1.5	1.7	2.4	2.8	3.1	3.7	4.4	
0.2	0.4	1.3	1.5	1.8	2.4	2.9	3.2	3.8	4.5	
0.2	0.4	1.3	1.5	1.8	2.5	2.9	3.2	3.9	4.5	
0.3	1	1.3	1.6	1.9	2.6	2.9	3.2	4	4.6	

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0.3	1	1.4	1.6	2	2.6	2.9	3.3	4	4.6
0.3	1.2	1.4	1.6	2	2.6	3	3.4	4	4.7

TABLE.2. Parameters Estimates and Goodness – of – Fits by akaike information criterion (AIC), akaike information criterion corrected (AICC)and Bayesakaike information criterion(BIC) .

Distributions	MLE	-2Ln log	AIC	AIC_C	BIC
Transmuted kappa Distribution	$\hat{\alpha} = 1.42777$ $\hat{\beta} = 1.285$ $\hat{\theta} = 4.95419$ $\hat{\gamma} = 0.471$	291.5943	299.399	299.8030	299.662
kappa Distribution	$\hat{\alpha} = 2.4532$ $\hat{\beta} = 4.8333$ $\hat{\theta} = 0.9309$	572.204	578.204	578.444	578.255
Linley Distribution	$\hat{\alpha} = 0.3828$ $\hat{\beta} = 1.5562$ $\hat{\theta} = 0.7988$	362.306	368.306	368.546	368.357
Gamma Distribution	$\hat{\alpha} = 1.73129$ $\hat{\beta} = 1.32738$	365.072	369.072	369.1908	369.106
Weibull Distribution	$\hat{\alpha} = 1.55908$ $\hat{\beta}$	356.936	360.936	361.0548	380.97006
E	$\hat{\beta}$ =	381.017	383.017	383.05621	383.034

5. CONCLUSION

In This paper, The New transformed Kappa Distribution (TKD),Some of The Properties are Derived and Discussed like Moments, Reliability Analysis, and Hazard rate. The Method of

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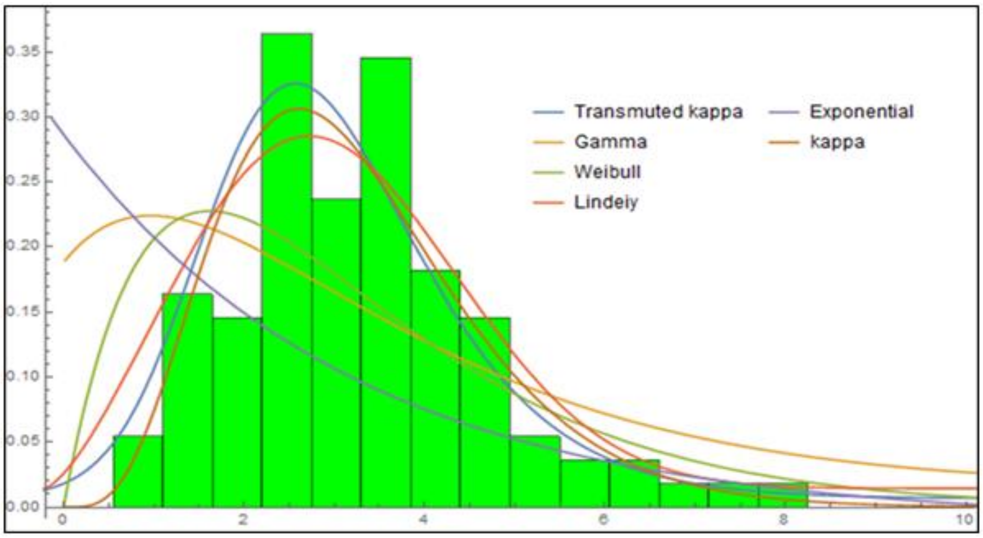
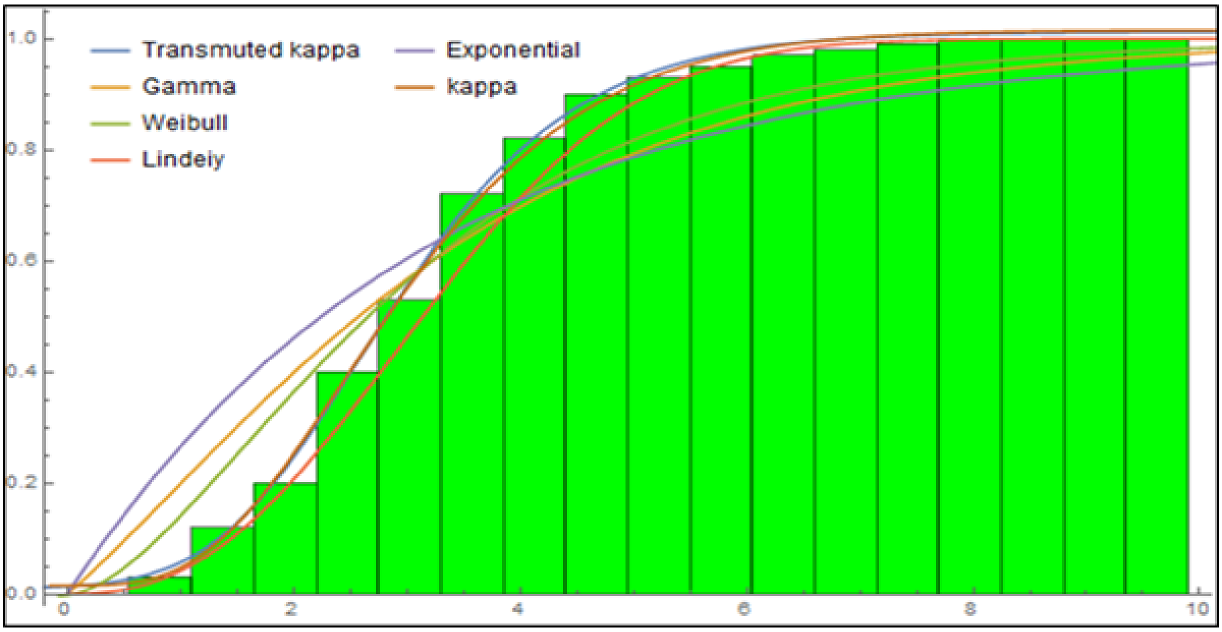
Maximum Likelihood Estimation is Used for Determining The parameters. The Berformance of The New Model is Determined By Fitting to real-life Data Using the Goodness of Fit Criteria Such as AIC, AIC_C,andBIC. ,The Appropriateness of The Real Data for probability Distributions Under Study It is Found transformed Kappa Distribution (TKD)gives a better fit to the data set as Compared One parameter Exponential Distribution (ED), Three Parameter lindely Distribution(TPLD), Gamma Distribution(G.D), and Three Parameter kappa Distribution (KD). and Weibull Distribution(WD),Depending on The values of AIC, AIC_CandBIC in the table (2).





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