JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 3590-3596
https://publishoa.com
ISSN: 1309-3452

Analysis on Retrial Queues with Hexagonal Fuzzy Parameters

S. Josephine Vinnarasi
Department of Mathematics, Holy Cross College (Autonomous),

Tiruchirappalli – 620 002, Tamilnadu, India.

Email : josesureshvasan@gmail.com

Received 2022 March 15; Revised 2022 April 20; Accepted 2022 May 10.

ABSTRACT

Fuzzy Queuing paradigm marks its productivity as an imperishableidiosyncrasy to forecast real world strategies with natural imprecise data. This paperdevelops the membership functions ofthe performance indicators entry, retrial and exit fashion of customers in retrial fuzzy queues incline to parametric non-linear approach on fuzzy grounds which is methodical, factual and is more functional for managers and system architects. A numerical illustration is exemplified to depict the efficacy of the proposed model.

Keywords: FuzzyQueues, Non-linear programming, Hexagonal fuzzy numbers, Retrial Queues.

I. INTRODUCTION

Fuzzy sets predict operating with linguistic terms, characterizing indefinite structures, handling ambiguous constraints,has risen as a new means of coping with unpredictability and perceptions, a sprout from lack of undefined peripheries, more utility than crisp information, has higher flexibility to reflect inherent vague concepts, has covered a variety of present world scenarios with great wonders. Fuzzy queuing is a modern strategy in real-world,its presence will assist the managers in facilitating service time in the face of uncertainty and maximizes benefits, insightful, much practical,facilitates adaptability for uncertain circumstances.

Retrial queues are scheduling devices/ customers that enable arriving customers who find both servers and waiting positions occupied to retry their request for service after some duration or randomly join later. A primary service network and an orbit make up a retrial queuing network. Customers are delivered to the service center at a Poisson rate from the main arena. Arrivals who discover the server is busy, under repair, or closed for holiday enter the retrial queue (orbit) to try again later or exit the distribution Centre instantaneously. This is commonly observed in restaurants, networking sectors, hospitals. It involves reservicing of customers when congestions are there mainly split as blocking and delay procedure that permits multiple trials.

Retrial Queues was studied long ago since 1957, several papers are widely researched in varied forms by many investigators like Falin [9], Yang[16], Diamond and Alpha [7], Artalejo J. R.,[1],Kulkani V.G., Liang H.M.,[12], Jau-Chaun Ke, Hsin-I Huang, Chuen-Horng Lin[10]elaborated on fuzzified customer arrival, retrial and service frame using parametric non-linear programs. This paper explainsthe membership procedures of the expected waiting time and customers in the orbit using NLP technique with hexagonal fuzzy numbersopting Yager's ranking index as a decision-making tool for optimization under uncertain environs.

II. FUZZY MODEL CONSTRUCTION

Assume FM/FM/1-(FR) model with entry rate $\overline{\lambda}_{AR}$ where the customers arrive for service, acquires service if the system is free, if found busy, he enters the orbit and retries for service after some period, called retrial time. The quantum of orbit is finite. They follow exponential distribution with fuzzy retrial rate $\overline{\theta}_{RT}$ and fuzzy service rate $\overline{\mu}_{SR}$. The arrival rate $\overline{\lambda}_{AR}$, retrial rate $\overline{\theta}_{RT}$, and service rate $\overline{\mu}_{SR}$ are assumed with its corresponding membership

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 3590-3596
https://publishoa.com
ISSN: 1309-3452

functions, making use of Zadeh's groundwork, as $\bar{\lambda}_{AR}=\left\{\left(x_{AR},\phi_{\bar{\lambda}_{AR}}\left(x_{AR}\right)\right)\middle|x_{AR}\in X\right\}$, $\bar{\theta}_{RT}=\left\{\left(v_{RT},\phi_{\bar{\theta}_{RT}}\left(v_{RT}\right)\right)\middle|v_{RT}\in V\right\}$; $\bar{\mu}_{SR}=\left\{\left(y_{SR},\phi_{\bar{\mu}_{SR}}\left(y_{SR}\right)\right)\middle|y_{SR}\in Y\right\}$

The system performance measure is defined as

$$\bar{\phi}_{f\left(\bar{\lambda}_{AR},\bar{\theta}_{RT},\bar{\mu}_{SR}\right)}(z)=\underset{x_{AR}\in X,v_{RT}\in V,y_{SR}\in Y}{supremum}\ minimum\left\{\phi_{\bar{\lambda}_{AR}}\left(x_{AR}\right),\phi_{\bar{\theta}_{RT}}\left(v_{RT}\right),\phi_{\bar{\mu}_{SR}}\left(y_{SR}\right)\middle|z=f\left(x_{AR},v_{RT},y_{SR}\right)\right\}$$

Assume the performance measures average waiting time and average number of consumers in the orbit, with $\rho=\frac{x_{AR}}{Y_{SR}}$, acquiring the knowledge of conventional retrial queues, we obtain $E_{MWT}\left(\bar{W}\right)=\frac{x_{AR}}{y_{SR}-x_{AR}}\left(\frac{1}{x_{AR}}+\frac{1}{v_{RT}}\right)$ and $E_{MWT}\left(\bar{N}\right)=\frac{x_{AR}^2}{y_{SR}-x_{AR}}\left(\frac{1}{x_{AR}}+\frac{1}{v_{RT}}\right)$.

A nonlinear parametric programming technique based on alpha cuts and extension principle thecalculations are performed.

$$\phi_{E_{MWT}\left(\bar{W}\right)}(z)=\underset{x_{AR}/y_{SR}<1}{supremum}\ minimum\left\{\phi_{\bar{\lambda}_{AR}}\left(x_{AR}\right),\phi_{\bar{\theta}_{RT}}\left(v_{RT}\right),\phi_{\bar{\mu}_{SR}}\left(y_{SR}\right)\middle|z=\frac{x_{AR}}{y_{SR}-x_{AR}}\left(\frac{1}{x_{AR}}+\frac{1}{v_{RT}}\right)\right.$$

$$\boldsymbol{\phi}_{\boldsymbol{E_{MCO}(\bar{N})}}(\boldsymbol{z})=\underset{\frac{x_{AR}}{y_{SR}}<1}{\boldsymbol{supremum}}\ minimum\left\{\phi_{\bar{\lambda}_{AR}}\left(x_{AR}\right),\phi_{\bar{\theta}_{RT}}\left(v_{RT}\right),\phi_{\bar{\mu}_{SR}}\left(y_{SR}\right)\middle|z=\frac{x_{AR}^2}{y_{SR}-x_{AR}}\left(\frac{1}{x_{AR}}+\frac{1}{v_{RT}}\right)\right.$$

By making use of Zadeh's groundwork, we define alpha-cuts as

$$\bar{\lambda}_{AR}(\alpha)=\left[x_{AR}^{LB},x_{AR}^{UB}\right]=\left[\underset{x_{AR}\in X}{minmium}\left\{x_{AR}\middle|\phi_{\bar{\lambda}_{AR}}\left(x_{AR}\right)\geq\alpha\right\},\underset{x_{AR}\in X}{maximum}\left\{x_{AR}\middle|\phi_{\bar{\lambda}_{AR}}\left(x_{AR}\right)\geq\alpha\right\}\right]$$

$$\bar{\theta}_{RT}(\alpha)=\left[v_{RT}^{LB},v_{RT}^{UB}\right]=\left[\underset{v_{RT}\in V}{minmium}\left\{v_{RT}\middle|\phi_{\bar{\theta}_{RT}}\left(v_{RT}\right)\geq\alpha\right\},\underset{v_{RT}\in V}{maximum}\left\{v_{RT}\middle|\phi_{\bar{\theta}_{RT}}\left(v_{RT}\right)\geq\alpha\right\}\right]$$

$$\bar{\mu}_{SR}(\alpha)=\left[y_{SR}^{LB},y_{SR}^{UB}\right]=\left[\underset{y_{SR}\in Y}{minmium}\left\{y_{SR}\middle|\phi_{\bar{\mu}_{SR}}\left(y_{SR}\right)\geq\alpha\right\},\underset{y_{SR}\in Y}{maximum}\left\{y_{SR}\middle|\phi_{\bar{\mu}_{SR}}\left(y_{SR}\right)\geq\alpha\right\}\right]$$

where the lower and upper bounds are obtained as

$$x_{\alpha}^{LB}=minimum\ \phi_{\bar{\lambda}_{AR}}^{-1}(\alpha),x_{\alpha}^{UB}=maximum\ \phi_{\bar{\lambda}_{AR}}^{-1}(\alpha),v_{\alpha}^{LB}=minimum\ \phi_{\bar{\theta}_{AR}}^{-1}(\alpha),$$

$$v_{\alpha}^{UB}=maximum\ \phi_{\bar{\theta}_{RT}}^{-1}(\alpha),y_{\alpha}^{LB}=minimum\ \phi_{\bar{\mu}_{RT}}^{-1}(\alpha),y_{\alpha}^{UB}=maximum\ \phi_{\bar{\mu}_{RT}}^{-1}(\alpha),$$

The membership functions are extracted when one of the following conditions are satisfied for $z=\frac{x_{AR}}{y_{SR}-x_{AR}}\left(\frac{1}{x_{AR}}+\frac{1}{v_{RT}}\right)$ satisfies $\phi_{E_{MWT}\left(\bar{W}\right)}(z)=\alpha$

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 3590-3596
https://publishoa.com
ISSN: 1309-3452

case (i): $\left(\phi_{\bar{\lambda}_{AR}}\left(x_{AR}\right)=\alpha,\phi_{\bar{\theta}_{RT}}\left(v_{RT}\right)\geq\alpha,\phi_{\bar{\mu}_{SR}}\left(y_{SR}\right)\geq\alpha\right),$

case (ii): $\left(\phi_{\bar{\lambda}_{AR}}\left(x_{AR}\right)\geq\alpha,\phi_{\bar{\theta}_{RT}}\left(v_{RT}\right)=\alpha,\phi_{\bar{\mu}_{SR}}\left(y_{SR}\right)\geq\alpha\right),$

case (iii): $\left(\phi_{\bar{\lambda}_{AR}}\left(x_{AR}\right)\geq\alpha,\phi_{\bar{\theta}_{RT}}\left(v_{RT}\right)\geq\alpha,\phi_{\bar{\mu}_{SR}}\left(y_{SR}\right)=\alpha\right),$

By parametric NLP technique the lower and upper bounds of the α-cut are:

Case (i)

$$\left(E_{MWT}\left(\bar{W}\right)\right)_{\alpha}^{LB_1}=\underset{x_{AR}/y_{SR}<1}{minimum}\left[\frac{x_{AR}}{y_{SR}-x_{AR}}\left(\frac{1}{x_{AR}}+\frac{1}{v_{RT}}\right)\right],$$

$$\left(\boldsymbol{E_{MWT}}(\bar{\boldsymbol{W}})\right)_{\boldsymbol{\alpha}}^{\boldsymbol{UB_1}}=\underset{\frac{x_{AR}}{y_{SR}}<1}{\boldsymbol{maximum}}\left[\frac{\boldsymbol{x_{AR}}}{\boldsymbol{y_{SR}}-\boldsymbol{x_{AR}}}\left(\frac{\mathbf{1}}{\boldsymbol{x_{AR}}}+\frac{\mathbf{1}}{\boldsymbol{v_{RT}}}\right)\right]$$

Case (ii)

$$\left(E_{MWT}\left(\bar{W}\right)\right)_{\alpha}^{LB_2}=\underset{x_{AR}/y_{SR}<1}{minimum}\left[\frac{x_{AR}}{y_{SR}-x_{AR}}\left(\frac{1}{x_{AR}}+\frac{1}{v_{RT}}\right)\right],$$

$$\left(\boldsymbol{E_{MWT}}(\bar{\boldsymbol{W}})\right)_{\boldsymbol{\alpha}}^{\boldsymbol{UB_2}}=\underset{\frac{x_{AR}}{y_{SR}}<1}{\boldsymbol{maximum}}\left[\frac{\boldsymbol{x_{AR}}}{\boldsymbol{y_{SR}}-\boldsymbol{x_{AR}}}\left(\frac{\mathbf{1}}{\boldsymbol{x_{AR}}}+\frac{\mathbf{1}}{\boldsymbol{v_{RT}}}\right)\right]$$

Case (iii)

$$\left(E_{MWT}\left(\bar{W}\right)\right)_{\alpha}^{LB_3}=\underset{x_{AR}/y_{SR}<1}{minimum}\left[\frac{x_{AR}}{y_{SR}-x_{AR}}\left(\frac{1}{x_{AR}}+\frac{1}{v_{RT}}\right)\right],$$

$$\left(\boldsymbol{E_{MWT}}(\bar{\boldsymbol{W}})\right)_{\boldsymbol{\alpha}}^{\boldsymbol{UB_3}}=\underset{\frac{x_{AR}}{y_{SR}}<1}{\boldsymbol{maximum}}\left[\frac{\boldsymbol{x_{AR}}}{\boldsymbol{y_{SR}}-\boldsymbol{x_{AR}}}\left(\frac{\mathbf{1}}{\boldsymbol{x_{AR}}}+\frac{\mathbf{1}}{\boldsymbol{v_{RT}}}\right)\right]$$

If both $\left(E_{MWT}\left(\bar{W}\right)\right)_{\alpha}^{LB}\left(E_{MWT}\left(\bar{W}\right)\right)_{\alpha}^{UB}$ are invertible with respect to α, then a left shape function $L\left(z\right)=\left[\left(E_{MWT}\left(\bar{W}\right)\right)_{\alpha}^{LB}\right]^{-1}$ and a right shape function $R\left(z\right)=\left[\left(E_{MWT}\left(\bar{W}\right)\right)_{\alpha}^{UB}\right]^{-1}$ are obtained from which the membership function is

$$\phi_{E_{MWT}(\bar{W})}\left(z\right)=\begin{cases}L\left(z\right), & \left(E_{MWT}\left(\bar{W}\right)\right)_{\alpha=0}^{LB}\leq z\leq\left(E_{MWT}\left(\bar{W}\right)\right)_{\alpha=1}^{LB},\\ 1, & \left(E_{MWT}\left(\bar{W}\right)\right)_{\alpha=1}^{LB}\leq z\leq\left(E_{MWT}\left(\bar{W}\right)\right)_{\alpha=1}^{UB},\\ R\left(z\right), & \left(E_{MWT}\left(\bar{W}\right)\right)_{\alpha=1}^{UB}\leq z\leq\left(E_{MWT}\left(\bar{W}\right)\right)_{\alpha=0}^{UB}\end{cases}$$

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 3590-3596
https://publishoa.com
ISSN: 1309-3452
In view of the system components, exhibited by membership functions, the parameters conserve fuzziness. But the organizations or consultants incline towards crisp value than fuzziness. To rectify such issues, defuzzification is done for the performance indicators by Yager's ranking index estimated as: $Yag\left(RI\right)=\int_0^1 \frac{\Omega_\alpha^{LB}+\Omega_\alpha^{UB}}{2}$

III. NUMERICAL EXPLORATION

Fuzzy Mean Waiting time in the Queue $E_{MWT}\left(\bar{W}\right)$

Assume arrival, retrial and service rates as hexagonal fuzzy numbers $\bar{\lambda}_{AR}=\left[4,5,6,7,8,9\right]$; $\bar{\theta}_{RT}=\left[2,9,16,23,30,37\right]$; $\bar{\mu}_{SR}=\left[10,11,12,13,14,15\right]$

The alpha-cuts are calculated as $\left[x_{AR}^{LB},x_{AR}^{UB}\right]=\left[4+2\alpha,9-2\alpha\right]$; $\left[v_{RT}^{LB},v_{RT}^{UB}\right]=\left[2+14\alpha,37-14\alpha\right]$ $\left[y_{SR}^{LB},y_{SR}^{UB}\right]=\left[10+2\alpha,15-2\alpha\right]$. Clearly when $x=x_\alpha^{UB},v=v_\alpha^{LB}$ and $y=y_\alpha^{LB}$, the expected waiting time in the orbit attains its maximum value, and when $x=x_\alpha^{LB},v=v_\alpha^{UB}$ and $y=y_\alpha^{UB}$ the expected waiting time in the orbit attains its minimum value.

$$\left(E_{MWT}\left(\bar{W}\right)\right)_\alpha^{LB}=\frac{-12\alpha+41}{56\alpha^2-302\alpha+407}\ ;\ \left(E_{MWT}\left(\bar{W}\right)\right)_\alpha^{UB}=\frac{12\alpha+11}{56\alpha^2+22\alpha+2}$$

The membership function is

$$\phi_{E_{MWT}(\bar{W})}\left(z\right)=\begin{cases}\dfrac{\left(-12+302z\right)-\sqrt{144+1936z+36z^2}}{112z}, & \dfrac{41}{407}\le z\le\dfrac{29}{161}\\ 1, & \dfrac{29}{161}\le z\le\dfrac{80}{23}\\ \dfrac{\left(12-22z\right)+\sqrt{36z^2+1936z+144}}{112z}, & \dfrac{80}{23}\le z\le\dfrac{77}{14}\end{cases}$$

Fuzzy Mean number of customers in the orbit $E_{MCO}\left(\bar{N}\right)$

$$\left(E_{MCO}\left(\bar{N}\right)\right)^{LB}=\frac{-24\alpha^2+34\alpha+164}{56\alpha^2-302\alpha+407};\ \left(E_{MCO}\left(\bar{N}\right)\right)_\alpha^{UB}=\frac{-24\alpha^2+86\alpha+99}{56\alpha^2+22\alpha+2}$$

The membership function is

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 3590-3596
https://publishoa.com
ISSN: 1309-3452

$$\phi_{E_{MCO}(\bar{N})}(z)=\begin{cases}\dfrac{(34+302z)-\sqrt{36z^2+18200z+16900}}{112z+48}, & \dfrac{164}{407}\le z\le\dfrac{174}{161}\\ 1, & \dfrac{174}{161}\le z\le\dfrac{161}{80}\\ \dfrac{(86-22z)+\sqrt{36z^2+18200z+16900}}{112z+48}, & \dfrac{161}{80}\le z\le\dfrac{692}{14}\end{cases}$$

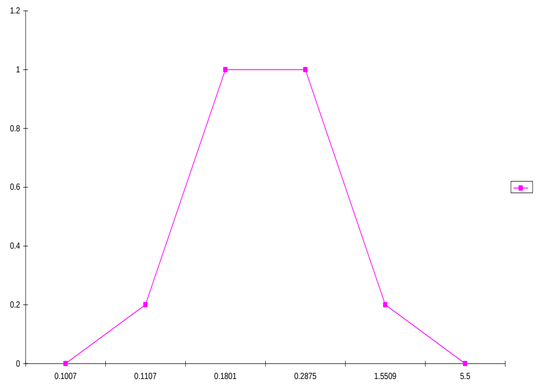
By Yager's Ranking Index,

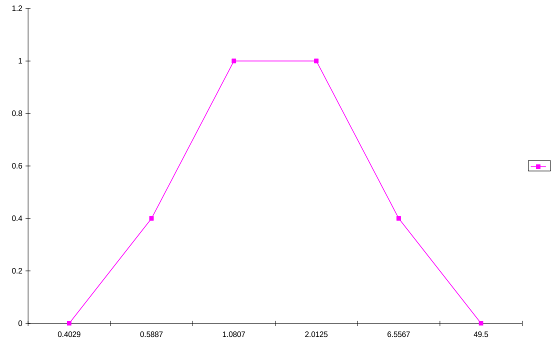
$$YRI\left(E_{MWT}(\bar{W})\right)=\int_0^1\frac{1}{2}\left[\frac{-12\alpha+41}{56\alpha^2-302\alpha+407}+\frac{12\alpha+11}{56\alpha^2+22\alpha+2}\right]d\alpha$$

$$=0.6026$$

$$YRI\left(E_{MCO}(\bar{N})\right)_\alpha^{UB}=\int_0^1\frac{1}{2}\left[\frac{-24\alpha^2+34\alpha+164}{56\alpha^2-302\alpha+407}+\frac{-24\alpha^2+86\alpha+99}{56\alpha^2+22\alpha+2}\right]d\alpha$$

$$=4.8734$$





Fuzzy mean waiting time **Fuzzy mean number of**

in the orbitcustomers in the orbit

The alpha cuts for arrival, retrial and service rates with mean waiting time in the orbit

Alpha	x_{AR}^{LB}	x_{AR}^{UB}	v_{RT}^{LB}	v_{RT}^{UB}	y_{SR}^{LB}	y_{SR}^{UB}	$\left(E_{MWT}(\bar{W})\right)_\alpha^{LB}$	$\left(E_{MWT}(\bar{W})\right)_\alpha^{UB}$
0.00	4.00	9.00	2.00	37.00	10.00	15.00	0.1007	5.5000
0.10	4.20	8.80	3.40	35.60	10.20	14.80	0.1055	2.5630
0.20	4.40	8.60	4.80	34.20	10.40	14.60	0.1107	1.5509
0.30	4.60	8.40	6.20	32.80	10.60	14.40	0.1164	1.0704
0.40	4.80	8.20	7.60	31.40	10.80	14.20	0.1226	0.7996

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 3590-3596
https://publishoa.com
ISSN: 1309-3452

0.50	5.00	8.00	9.00	30.00	11.00	14.00	0.1296	0.6296
0.60	5.20	7.80	10.40	28.60	11.20	13.80	0.1374	0.5147
0.70	5.40	7.60	11.80	27.20	11.40	13.60	0.1462	0.4326
0.80	5.60	7.40	13.20	25.80	11.60	13.40	0.1560	0.3716
0.90	5.80	7.20	14.60	24.40	11.80	13.20	0.1673	0.3246
1.00	6.00	7.00	16.00	23.00	12.00	13.00	0.1801	0.2875

The alpha cuts for the performance measures are obtained for distinct values of α. The crisp intervals for fuzzy waiting time at varied α levels are tabulated. $E_{MWT}\left(\bar{W}\right)$ extends from 0.1007 to 5.5 which predicts that though the expected waiting duration is fuzzy, it is not possible for the values to fall within 0.1007 or go beyond 5.5 when the alpha cut is 1, we obtain from 0.1081 to 0.2875, which interprets the suitable possible value for the average waiting time in the orbit.

The alpha cuts for arrival, retrial and service rates with mean number of customers in the orbit

Alpha	x_{AR}^{LB}	x_{AR}^{UB}	v_{RT}^{LB}	v_{RT}^{UB}	y_{SR}^{LB}	y_{SR}^{UB}	$\left(E_{MCO}\left(\bar{N}\right)\right)_{\alpha}^{LB}$	$\left(E_{MCO}\left(\bar{N}\right)\right)_{\alpha}^{UB}$
0.00	4.00	9.00	2.00	37.00	10.00	15.00	0.4029	49.5000
0.10	4.20	8.80	3.40	35.60	10.20	14.80	0.4430	22.5546
0.20	4.40	8.60	4.80	34.20	10.40	14.60	0.4869	13.3380
0.30	4.60	8.40	6.20	32.80	10.60	14.40	0.5352	8.9912
0.40	4.80	8.20	7.60	31.40	10.80	14.20	0.5887	6.5567
0.50	5.00	8.00	9.00	30.00	11.00	14.00	0.6481	5.0370
0.60	5.20	7.80	10.40	28.60	11.20	13.80	0.7146	4.0147
0.70	5.40	7.60	11.80	27.20	11.40	13.60	0.7893	3.2881
0.80	5.60	7.40	13.20	25.80	11.60	13.40	0.8738	2.7496
0.90	5.80	7.20	14.60	24.40	11.80	13.20	0.9701	2.3371
1.00	6.00	7.00	16.00	23.00	12.00	13.00	1.0807	2.0125

From the above table, the fuzzy average number of customers in the orbit for the different in the orbit for the different values of alpha are tabulated. $E_{MCO}\left(\bar{N}\right)$ falls in the interval [1.0807, 2.0125] for α=1 and we get for α=0 as [0.4029, 49.50] which indicates that the average number of customers will not extend above 49.50 or fall below 0.4029.

This numerical study reaps the benefit and captures perception for the possible average customers in the orbit.

IV. CONCLUSION

Fuzzy waiting line models are more adequate than the ordinary queues in our day-to-day realistic circumstances. This work projects the membership functions of the parameters adopting alpha cut and Zadeh's approach with Parametric non-linear programming technique.

From the calculated data, we can attain the range of waiting time with α level 0.8 as [0.1560, 0.3716] with $y_{\alpha}^{LB}=11.60$ and $y_{\alpha}^{UB}=13.40$ The range of customers with α level 0.5 as [0.6481, 5.0370] with $y_{\alpha}^{LB}=11.00$ and $y_{\alpha}^{UB}=14.00$

The designed prototype serves system organizers and practitioners with practicability and utility in real world environs.

V. REFERENCES

1. Artalejo. J.R., (1999), “A classified bibliography of research on retrial queues”: Progress in 1990-1999, Top 7, pp. 187-211.
2. Artalejo. J.R.,(1999),“Accessible bibliography on retrial queues”, Mathematical and Computer Modelling, Volume 30,pp.1-6
3. Artalejo. J.R., M. J. Lopez-Herrero, (2000), “On the single server retrial queue with balking”, Information systems and operational research, 38 (1),pp. 33-50.
4. ChanasS., Nowakowski.M, (1988), “Single value simulation of fuzzy variable”, Fuzzy set. Syst. 21, pp. 43-57.
5. Chen S.P., (2004), “Parametric nonlinear programming for analyzing fuzzy queues with finite capacity”, European Journal of Operational Research, 157,pp. 429-438.
6. Chen S.P., (2005), “Parametric nonlinear programming approach to fuzzy queues with bulk service”, European Journal of Operational Research,163,pp. 434-444.
7. Diamond J.E., Alfa A.S., (1999), “Approximation method foe M/PH/1 retrial queues with phase type inter-retrial times”, European Journal of Operational Research,113, pp. 620-631
8. Dudin A.N., Krishamoorthy, Joshua V.C., Tsarenkov G.V., (2004), “Analysis of the BMAP/G/1 retrial system with search of customers from the orbit”, European Journal of Operational Research,157,pp. 169-179.
9. Falin G.I., Templeton J.G.C., (1997), “Retrial Queues”, Chapman and Hall, London.
10. Jau-Chuan Ke, Hsin-I Huang, Chuen-Horng Lin, (2007),“On retrial queuing model with fuzzy parameters”, Elsevier B.V, Physica A 374, pp. 272-280
11. Chiang Kao, Chang Chung Li, Shih Pin Chen (1999), “Parametric Programming to the analysis of fuzzy queues”, Fuzzy sets and Systems, Volume 107, Issue 1, pp. 93-100.
12. Kulkarni. V.G, H.M. Liang, (1997),Retrial queues revisited, in: J.H. Dshalalow (Ed.),Frontiers in Queueing: Models and Applications in Science and Engineering, CRC press, Inc,pp. 19-34.
13. Li. R.J., E.S. Lee., (1989), “Analysis of fuzzy queues”, Computers and Mathematics with Applications, 17(7),pp. 1143-1147.
14. Negi. D.S. E.S Lee,(1992), “Analysis and simulation of fuzzy queues”, Fuzzy sets and Systems, 46,pp. 321-330.
15. Rajarajeswari P., Sahaya Sudha A., and Karthika R., (2013), “A new operation on hexagonal fuzzy number”, International Journal of Fuzzy Logic Systems (IJFLS) Vol.3, No.3, pp. 15-26.
16. Yang. T, J.G.C. Templeton, “A survey on retrial queues”, Queueing Syst. 2 (1987), pp. 201-233.