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Alternate Methods to find Rank of Matrix

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Abstract: letA bea matrix of order mXn .Since we have traditional method to find the rank of matrix A by converting the given matrix into row reduced echelon form or column reduced echelon and normal form. Here we use another method to convert the given matrix A into an Anti –Row reduced Echelon form or Anti- Column Reduced Echelon form and the result matrix will give the rank of the matrix A so that the number of non-zero rows in corresponding method is the rank of A. And we have also introduced anAnti Normal Form to find the rank of A. In addition to this we have solved system of linear equations using anti echelon method.

Keywords: Anti Identity matrix, Anti Row reduced-Echelon Form, Anti Normal Form

1. Introduction

Definition 1.1: A square matrix J of order nXn is said to be Anti Identity matrix if J represented as

$$J_{nXn} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & \iddots & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The Anti-Identity matrix of the different order for n=2, 3, 4, 5,., .are given below.

$$J_{2X2} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad J_{3X3} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \quad J_{4X4} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} ..$$

Definition 1.2: A matrix of of order mXn is said to be in Anti- Row reduced echelon form if it satisfy the following conditions

1. The last non-zero entry in first row is must be non-zero.
2. In each row below the last non-zero element below values must be zero.
3. Zero rows if any must be in bottom of the matrix.

Examples:

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 4 & 2 & 0 \\ 2 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 4 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 3 & 4 \\ 2 & 4 & 5 & 0 \\ 2 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 3 & 4 \\ 2 & 4 & 5 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Definition 1.3:A matrix of order mXn is said to be in Anti- Normal form if it is converted into any one the following form

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$[J], \begin{bmatrix} 0 & J \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} J \\ 0 \end{bmatrix}, [J \quad 0]$ Where J is Anti Identity matrix of order rXr and r is the rank and r ≤ Min (m, n)

For example

1) $\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$ is in anti normal form with rank 4

2) $\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ is in anti normal form with rank 3

3) $\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ is in anti normal form with rank 2

4) $\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ is in anti normal form with rank 2

Definition 1.4:A matrix of of order mXnis said to be in Row reduced echelon form if it satisfy the following conditions

1. The first non-zero entry in first row is must be non-zero.
2. In each row below the first non-zero element below values must be zero.
3. Zero rows if any must be in bottom of the matrix.

Examples:

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 3 & 4 \\ 0 & 4 & 5 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 3 & 4 \\ 0 & 4 & 5 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Definition 1.5:A matrix of order mXn is said to be in Normal form if it is converted into any one the following form

$[I], \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} I \\ 0 \end{bmatrix}, [I \quad 0]$Where r is I is the Identity matrix of order rXr and r is the rank, r ≤ Min (m, n)

For example

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1) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ is in anti normal form with rank 4

2) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ is in anti normal form with rank 3

3) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ is in anti normal form with rank 2

4) $\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ is in anti normal form with rank 2

II.Methodology

In this section we are going to take one example for this we will apply traditional method converting the given matrix into row reduced echelon form then we find rank of the matrix and then for the same matrix we will find the rank by converting the same matrix into anti row reduced echelon form. In the same way we will take another example to find the solution of the system of linear equationsconstruct augmented matrix and convert that into echelon form and Anti Echelon form to find the solution then we prove that using this two methods the answer is same.

Type 1: Row reduced Echelon form and Row reduced anti Echelon form

Case 1: Find Rank of a matrix using traditional method converting the given matrix into row reduced echelon form.

Find Rank of A= $\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & 7 \end{bmatrix}$ using row reduced echelon form

Solution:

$$A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$

R_1<->R_2

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$$\approx \begin{bmatrix} 1 & -1 & -2 & -4 \\ 2 & 3 & -1 & -1 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$

Applying $R_2\text{->}R_2\text{-}2R_1, R_3\text{->}R_3\text{-}3R_1, R_4\text{->}R_4\text{-}6R_1$

$$\approx \begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 5 & 3 & 7 \\ 0 & 4 & 9 & 10 \\ 0 & 9 & 12 & 17 \end{bmatrix}$$

Applying $R_4\text{->}R_4\text{-}R_3$

$$\approx \begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 5 & 3 & 7 \\ 0 & 4 & 9 & 10 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Applying $R_3\text{->}5R_4\text{-}4R_3$

$$\approx \begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 5 & 3 & 7 \\ 0 & 0 & 33 & 22 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

It is in echelon form now, the number of Non zero rows =3

$\therefore \rho(A) = 3$

Case 2: Find Rank of the same above matrix using alternative method converting the given matrix into anti row reduced echelon form

Find Rank of A= $\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & 7 \end{bmatrix}$ using Anti Row reduced echelon form

Solution:

$$A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$

Applying $R2\text{->}R_2\text{-}4R_1, R_3\text{->}R_3\text{-}2R_1, R_4\text{->}R_4\text{-}7R_1$

$$\approx \begin{bmatrix} 1 & -1 & -2 & -4 \\ -7 & -13 & 2 & 0 \\ -1 & -5 & 5 & 0 \\ -8 & -18 & 7 & 0 \end{bmatrix}$$

Applying $R_2\text{->}R_2\text{+}R_3$

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$$\approx \begin{bmatrix} 1 & -1 & -2 & -4 \\ -8 & -18 & 7 & 0 \\ -1 & -5 & 5 & 0 \\ -8 & -18 & 7 & 0 \end{bmatrix}$$

Applying $R_4 \to R_4 - R_2$

$$\approx \begin{bmatrix} 1 & -1 & -2 & -4 \\ -8 & -18 & 7 & 0 \\ -1 & -5 & 5 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Applying $R_3 \to 5R_3 - 7R_2$

$$\approx \begin{bmatrix} 1 & -1 & -2 & -4 \\ -1 & -5 & 5 & 0 \\ 33 & 55 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

It is in anti row reduced echelon form now, the number of Non zero rows =3

$\therefore \rho(A) = 3$

Hence the both cases the rank is unique

Type 2: Normal form and Anti Normal form

Case 1: Find Rank of a matrix using traditional method converting the given matrix into normal form.

Find Rank of A= $\begin{bmatrix} 1 & 2 & -2 & -3 \\ 2 & 5 & -4 & -6 \\ -1 & -3 & 2 & -2 \\ 2 & 4 & -1 & 6 \end{bmatrix}$ using row reduced Normal form

Solution:

$$A = \begin{bmatrix} 1 & 2 & -2 & -3 \\ 2 & 5 & -4 & -6 \\ -1 & -3 & 2 & -2 \\ 2 & 4 & -1 & 6 \end{bmatrix}$$

Applying $R_2 \to R_2 - 2R_1, R_3 \to R_3 + R_1, R_4 \to R_4 - 2R_1$

$$\approx \begin{bmatrix} 1 & 2 & -2 & -3 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & 3 & 0 \end{bmatrix}$$

Applying $C_2 \to C_2 - 2C_1, C_3 \to C_3 + 2C_1, C_4 \to C_4 + 3C_1$

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$$\approx \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & 3 & 0 \end{bmatrix}$$

Applying C_4<->$C_3$and then C_4->C_4/3

$$\approx \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\approx [I_4]$$

$$\therefore \rho(A) = 4$$

Case 2: Find Rank of the same above matrix using alternative method converting the given matrix into Anti Normal form

Solution:

$$A = \begin{bmatrix} 1 & 2 & -2 & -3 \\ 2 & 5 & -4 & -6 \\ -1 & -3 & 2 & -2 \\ 2 & 4 & -1 & 6 \end{bmatrix}$$

Applying R_2->R_2-2R_1,R_3->3R_3-2R_1,R_4->R_4+2R_1

$$\approx \begin{bmatrix} 1 & 2 & -2 & -3 \\ 0 & 1 & 0 & 0 \\ -5 & -13 & 10 & 0 \\ 4 & 8 & -7 & 0 \end{bmatrix}$$

Applying C_4->C_4/-3

$$\approx \begin{bmatrix} 1 & 2 & -2 & 1 \\ 0 & 1 & 0 & 0 \\ -5 & -13 & 10 & 0 \\ 4 & 8 & -7 & 0 \end{bmatrix}$$

Applying C_1->C_1-C_4,C_2->C_2-2C_4,C_3->C_3+2C_4

$$\approx \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ -5 & -13 & 10 & 0 \\ 4 & 8 & -7 & 0 \end{bmatrix}$$

C_4<->C_3

$$\approx \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ -5 & 10 & -13 & 0 \\ 4 & -7 & 8 & 0 \end{bmatrix}$$

Applying $R_3 \to R_3+13R_2, R_4 \to R_4-8R_2$

$$\approx \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ -5 & 10 & 0 & 0 \\ 4 & -7 & 0 & 0 \end{bmatrix}$$

Applying $R_3 \to R_3/-5$

$$\approx \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ -1 & 2 & 0 & 0 \\ 4 & -7 & 0 & 0 \end{bmatrix}$$

$C_2 \leftrightarrow C_1$

$$\approx \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 2 & -1 & 0 & 0 \\ -7 & 4 & 0 & 0 \end{bmatrix}$$

Applying $R_4 \to R_4+4R_3$,

$$\approx \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 2 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$C_2 \to C_2/-1$

$$\approx \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 2 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

Applying $C_1 \to C_1-2C_2$

$$\approx \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$$\approx [J_4]$$

$$\therefore \rho(A) = 4$$

Hence the both cases the rank is unique

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III. Conclusion

Here we have proved there are the alternative methods anti row reduced echelon form and Anti Normal form to find rank of the given matrix of any order.

Reference

1. Bourbaki, Algebra, ch. II, §10.12, p. 359