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Connectivity Indices of Labeled Signed Graphs Using Domination Degree

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Abstract

Using the concepts of minimal dominating sets,dominance degree and signed degree we define Randic and Sum connectivity indices for labeled signed graphs in this article. For several standard labeled signed graphs, we find various generalizations for these indices and explain the results that we have obtained.

AMS subject classification (2010): 05C22, 05C50.
Key words: Labeled signed graph, Domination degree, Signed degree, Connectivity indices.

1 Introduction

The topological indices[4] are graph invariants that provide us with internal information about the chemical molecule. Randic index and Sum connectivity index are the connectivity indices. Milan Randic created the Randic connectivity index in 1975 and Sum connectivity is the additive version of Milan Randic's connectedness index, which he introduced in 1975.

Definition 1.1 *The Randic connectivity index[1, 2], denoted by* $R(G)$*, is defined as*

$$R(G) = \sum_{w_i w_j \in E(G)} \frac{1}{\sqrt{d_i d_j}}$$

The Sum connectivity index[3], denoted by $S(G)$, is defined as

$$S(G) = \sum_{w_i w_j \in E(G)} \frac{1}{\sqrt{d_i + d_j}}$$

Definition 1.2 *A signed graph[5]* $\Sigma = (G, \sigma) = (W, E, \sigma)$ *is a graph which assigns a sign* $\sigma(e) \in \{+, -\}$ *to each vertex in* G*.* σ *is called signed function.*

Definition 1.3 *A set* $D \subseteq V$ *is said to be a dominating set[8] of G if there exists a vertex* $u \in D$ *such that u and v are adjacent to any vertex* $v \in V - D$*.*

Definition 1.4 *If* $D - V_i$ *is not a dominating set, the dominating set* $D = \{w_1, w_2, \cdots, w_r\}$ *is said to be minimum[8].*

Definition 1.5 *The dominance degree for any vertex[8]* $v \in V(G)$ *is denoted by* $d_d(v)$ *and is defined as the number of minimal dominating sets of G that contain v.*

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For more details of domination in graphs, refer to [6].

Definition 1.6 *Positive degree[7] is the sum of the membership values of all incident positive edges to* V .

$$deg^{+}\left[\sigma(v)\right] = \Sigma_{\mu^{+}(v,v_i)\in E}\mu^{+}(v,v_i).$$

Negative degree[7] is the sum of the membership values of all incident negative edges to V .

$$deg^{-}\left[\sigma(v) = \Sigma_{\mu^{-}(v,v_i)\in E}\mu^{-}(v,v_i)\right].$$

A vertex's signed degree[7] is the difference between its positive and negative degrees.

$$sdeg(v) = | deg^{+}\left[\sigma(v)\right] - deg^{-}\left[\sigma(v)\right] | .$$

2 Connecivity Indices for labeled signed graph using domination degree

In [8], Hanan Ahmed, Anwar Alwardi and Rubi Selestina Morgan have defined minimal dominating set and domination degree and has given domination degree for various standard graphs. Motivated by the same we define some connectivity indices of labeled signed graphs using domination degree for some standard graphs and find the generalizations for those indices.

Let G be a simple graph. The labeled signed graph of G using domination degree is obtained as using the following steps.

STEP 1: Label the vertex with the domination degree of the particular graph.
STEP 2: Assign positive sign to the vertex which is labeled with even degree.
STEP 3: Assign negative sign to the vertex which is labeled with odd degree.
STEP 4: Assign the signs to the edges with the signs of the product of the corresponding vertices.
STEP 5: Calculate the signed degree of each vertices in the graph G

We define the topological index for labeled signed graph using domination degree as follows.

Definition 2.1 *The Randic connectivity index for* $\mathrm{LS_G}$ *is denoted by* $RC(\mathrm{LS_G})$ *and is defined as*

$$RC(\mathrm{LS_G}) = \Sigma_{i,j=1}^{n} \frac{1}{\sqrt{d_d(w_i)\,(sdeg(w_i))\; d_d(v_j)\,(sdeg(v_j))}}.$$

Definition 2.2 *The Sum connectivity index for* $\mathrm{LS_G}$ *is denoted by* $SC(\mathrm{LS_G})$ *and is defined as*

$$SC(\mathrm{LS_G}) = \Sigma_{i,j=1}^{n} \frac{1}{\sqrt{d_d(w_i)\,(sdeg(w_i)) + d_d(v_j)\,(sdeg(v_j))}}.$$

2.1 Complete Labeled Signed Graph.

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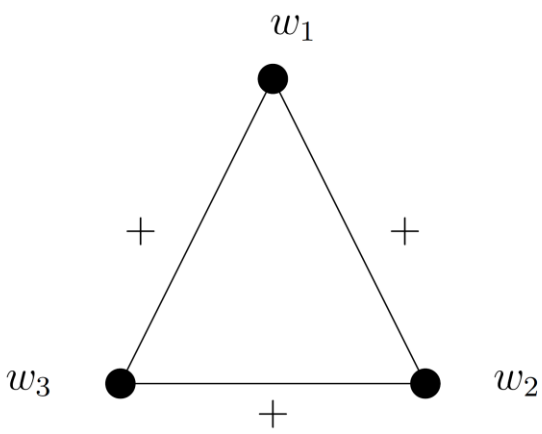

Figure 1: Complete Graph(K_n)

From the figure 1 given below, the domination degree for all the vertices for the complete graph is one. We label the vertices with negative sign since we have odd number. We label the edges with the product of the corresponding vertices.

$$sdeg(w_1) = 2,\ sdeg(w_2) = 2,\ sdeg(w_3) = 2.$$

$$\begin{aligned} RC(\mathrm{LS_G}) &= \Sigma_{i,j=1}^{n} \frac{1}{\sqrt{d_d(w_i)\,(sdeg(w_i))\,d_d(v_j)\,(sdeg(v_j))}} \\ &= \frac{1}{\sqrt{(1\times2\times1\times2)}} + \frac{1}{\sqrt{(1\times2\times1\times2)}} + \frac{1}{\sqrt{(1\times2\times1\times2)}} \\ &= 0.5+0.5+0.5 \\ &= 1.5 \end{aligned}$$

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$$\begin{aligned} SC(\mathrm{LS_G}) &= \Sigma_{i,j=1}^{n} \frac{1}{\sqrt{d_d(w_i)\,(sdeg(w_i)) + d_d(v_j)\,(sdeg(v_j))}} \\ &= \frac{1}{\sqrt{(1\times2+1\times2)}} + \frac{1}{\sqrt{(1\times2+1\times2)}} + \frac{1}{\sqrt{(1\times2+1\times2)}} \\ &= 0.5+0.5+0.5 \\ &= 1.5 \end{aligned}$$

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Theorem 2.1 *If* $LS(K_{1,n-1})$ *is the labeled signed star graph then the Randic connectivity index is given as* $RC(K_{1,n-1}) = \dfrac{n-1}{\sqrt{n-1}}$

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Proof. Let $LS(K_{1,n-1})$ be a labeled signed star graph.

Since the domination degree for the star graph is 1 for all the vertices, we label all the vertices of the star graph with one. Assign negative sign to all the vertices. Label the edges with the product of the corresponding vertices.

$\therefore$ Signed degree of u_1 is $(n-1)$ and signed degree of remaining vertices is 1. The Randic connectivity index of labeled signed star graph is

$$RC(K_{1,n-1}) = \Sigma_{i=1}^{n} \frac{1}{\sqrt{d_d(u_i)\,(sdeg(u_i))\,d_d(u_j)\,(sdeg(u_j))}}$$

$$= \frac{n-1}{\sqrt{1\,(n-1)\,(1)\,(1)}}$$

$$= \frac{n-1}{\sqrt{n-1}}$$

Theorem 2.2 *If* $LS(K_n)$ *is the labeled signed complete graph then the Randic connectivity index is given as* $RC(K_n) = \dfrac{n}{n-1}$.

Proof. Let $LS(K_n)$ be a labeled signed complete graph.

Since the domination degree for the complete graph is 1 for all the vertices, we label all the vertices of the complete graph with one. Assign negative sign to all the vertices. Label the edges with the product of the corresponding vertices.

$\therefore$ Signed degree of all these vertices is (n-1). The Randic connectivity index of labeled signed complete graph is

$$RC(K_n) = \Sigma_{i=1}^{n} \frac{1}{\sqrt{d_d(u_i)\,(sdeg(u_i))\,d_d(u_j)\,(sdeg(u_j))}}$$

$$= \frac{n}{\sqrt{1\,(n-1)\,1\,(n-1)}}$$

$$= \frac{n}{n-1}$$

Theorem 2.3 *If* $LS(DS_{r,n})$ *is the labeled signed double-star graph then the Randic connectivity index is given as* $RC(DS_{r,n}) = \dfrac{n-1}{\sqrt{n}} + \dfrac{1}{2n}$.

Proof. Let $LS(DS_{r,n})$ be a labeled signed double-star graph.

Since the domination degree for the double-star graph is 2 for all the vertices, we label all the vertices of the star graph with two. Assign positive sign to all the vertices. Label the edges with the product of the corresponding vertices.

$\therefore$ Signed degree of u_1 is $(n+1)$ and signed degree of remaining vertices is 1. The Randic

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connectivity index of labeled signed star graph is

$$RC(DS_{r,n}) = \Sigma_{i=1}^{n} \frac{1}{\sqrt{d_d(u_i)\,(sdeg(u_i))\,d_d(u_j)\,(sdeg(u_j))}}$$

$$= \frac{2n-2}{\sqrt{2\,(n)\,2\,(1)}} + \frac{1}{\sqrt{2\,(n)\,(2)\,(n)}}$$

$$= \frac{n-1}{\sqrt{n}} + \frac{1}{2n}$$

Theorem 2.4 *If* $LS(K_{m,n})$ *is the labeled signed complete bipartite graph with n being odd then the Randic connectivity index is given as* $RC(K_m, n) = \frac{1}{n+1}$.

Proof. Let $LS(K_{m,n})$ be a labeled signed complete bipartite graph with n as odd.

Since the domination degree for the complete bipartite graph is $d(v_i)+1$ for all the vertices, we label all the vertices of the complete bipartite graph according to this. Assign positive sign to all the vertices. Label the edges with the product of the corresponding vertices.

$\therefore$ Signed degree of all these vertices is n. The Randic connectivity index of labeled signed complete bipartite graph is

$$RC(K_{m,n}) = \Sigma_{i=1}^{n} \frac{1}{\sqrt{d_d(u_i)\,(sdeg(u_i))\,d_d(u_j)\,(sdeg(u_j))}}$$

$$= \frac{n}{\sqrt{n\,(n+1)\,n\,(n+1)}}$$

$$= \frac{1}{n+1}$$

Theorem 2.5 *If* $LS(K_{m,n})$ *is the labeled signed complete bipartite graph with n being even then the Randic connectivity index is given as* $RC(K_m, n) = \frac{1}{n+1}$.

Proof. Let $LS(K_{m,n})$ be a labeled signed complete bipartite graph with n as even.

Since the domination degree for the complete bipartite graph is $d(v_i)+1$ for all the vertices, we label all the vertices of the complete bipartite graph according to this. Assign positive sign to all the vertices. Label the edges with the product of the corresponding vertices.

$\therefore$ Signed degree of all these vertices is n. The Randic connectivity index of labeled signed complete bipartite graph is

$$RC(K_{m,n}) = \Sigma_{i=1}^{n} \frac{1}{\sqrt{d_d(u_i)\,(sdeg(u_i))\,d_d(u_j)\,(sdeg(u_j))}}$$

$$= \frac{n}{\sqrt{n\,(n+1)\,n\,(n+1)}}$$

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$$= \frac{1}{n+1}$$

Theorem 2.6 *If* $LS(K_{1,n-1})$ *is the labeled signed star graph then the Sum connectivity index is given as* $SC(K_{1,n-1}) = \frac{n-1}{\sqrt{n}}$.

Proof. Let $LS(K_{1,n-1})$ be a labeled signed star graph.

Since the domination degree for the star graph is 1 for all the vertices, we label all the vertices of the star graph with one. Assign negative sign to all the vertices. Label the edges with the product of the corresponding vertices.

$\therefore$ Signed degree of u_1 is $(n-1)$ and signed degree of remaining vertices is 1. The Sum connectivity index of labeled signed star graph is

$$SC(K_{1,n-1}) = \Sigma_{i=1}^{n} \frac{1}{\sqrt{d_d(u_i)\,(sdeg(u_i)) + d_d(u_j)\,(sdeg(u_j))}}$$

$$= \frac{n-1}{\sqrt{1\,(n-1)+(1)\,(1)}}$$

$$= \frac{n-1}{\sqrt{n}}$$

Theorem 2.7 *If* $LS(K_n)$ *is the labeled signed complete graph then the Sum connectivity index is given as* $SC(K_n) = \frac{n}{\sqrt{2\,n-1}}$.

Proof. Let $LS(K_n)$ be a labeled signed complete graph.

Since the domination degree for the complete graph is 1 for all the vertices, we label all the vertices of the complete graph with one. Assign negative sign to all the vertices. Label the edges with the product of the corresponding vertices.

$\therefore$ Signed degree of all these vertices is (n-1). The Sum connectivity index of labeled signed complete graph is

$$SC(K_n) = \Sigma_{i=1}^{n} \frac{1}{\sqrt{d_d(u_i)\,(sdeg(u_i)) + d_d(u_j)\,(sdeg(u_j))}}$$

$$= \frac{n}{\sqrt{1\,(n-1)+1\,(n-1)}}$$

$$= \frac{n}{\sqrt{2\,n-1}}$$

Theorem 2.8 *If* $LS(DS_{r,n})$ *is the labeled signed double-star graph then the Sum connectivity*

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index is given as $SC(DS_{r,n}) = \frac{n-1}{\sqrt{n}} + \frac{1}{2n}$.

Proof. Let $LS(DS_{r,n})$ be a labeled signed double-star graph.

Since the domination degree for the double-star graph is 2 for all the vertices, we label all the vertices of the star graph with two. Assign positive sign to all the vertices. Label the edges with the product of the corresponding vertices.

$\therefore$ Signed degree of u_1 is $(n+1)$ and signed degree of remaining vertices is 1. The Sum connectivity index of labeled signed star graph is

$$SC(DS_{r,n}) = \Sigma_{i=1}^{n} \frac{1}{\sqrt{d_d(u_i)\,(sdeg(u_i)) + d_d(u_j)\,(sdeg(u_j))}}$$

$$= \frac{2n-2}{\sqrt{2\,(n) + 2\,(1)}} + \frac{1}{\sqrt{2\,(n) + (2)\,(n)}}$$

$$= \frac{2n-2}{\sqrt{2(n+1)}} + \frac{1}{2\sqrt{n}}$$

Theorem 2.9 *If* $LS(K_{m,n})$ *is the labeled signed complete bipartite graph with n being odd then the Sum connectivity index is given as* $SC(K_{m,n}) = \frac{n}{\sqrt{2(n^2+1)}}$.

Proof. Let $LS(K_{m,n})$ be a labeled signed complete bipartite graph with n as odd.

Since the domination degree for the complete bipartite graph is $d(v_i)+1$ for all the vertices, we label all the vertices of the complete bipartite graph according to this. Assign positive sign to all the vertices. Label the edges with the product of the corresponding vertices.

$\therefore$ Signed degree of all these vertices is n. The Sum connectivity index of labeled signed complete bipartite graph is

$$SC(K_{m,n}) = \Sigma_{i=1}^{n} \frac{1}{\sqrt{d_d(u_i)\,(sdeg(u_i)) + d_d(u_j)\,(sdeg(u_j))}}$$

$$= \frac{n}{\sqrt{n\,(n+1) + n\,(n+1)}}$$

$$= \frac{n}{\sqrt{2(n^2+1)}}$$

Theorem 2.10 *If* $LS(K_{m,n})$ *is the labeled signed complete bipartite graph with n being even then the Sum connectivity index is given as* $SC(K_m, n) = \frac{n}{\sqrt{2(n^2+1)}}$.

Proof. Let $LS(K_{m,n})$ be a labeled signed complete bipartite graph with n as even.

Since the domination degree for the complete bipartite graph is $d(v_i)+1$ for all the

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vertices, we label all the vertices of the complete bipartite graph according to this. Assign positive sign to all the vertices. Label the edges with the product of the corresponding vertices.

$\therefore$ Signed degree of all these vertices is n. The Sum connectivity index of labeled signed complete bipartite graph is

$$SC(K_{m,n}) = \Sigma_{i=1}^{n} \frac{1}{\sqrt{d_d(u_i)\,(sdeg(u_i)) + d_d(u_j)\,(sdeg(u_j))}}$$

$$= \frac{n}{\sqrt{n\,(n+1) + n\,(n+1)}}$$

$$= \frac{n}{\sqrt{2(n^2+1)}}$$

Since the signed degree of the vertices other than the centre vertex is zero for windmill graph, the Randic connectivity and Sum connectivity indices value is also zero.

3 Conclusion

Labeled signed graphs on the molecular descriptors Randic connectivity and Sum connectivity indices are defined using domination degree,signed degree and have discussed with illustration for different types of standard signed graphs. The generalizations to find these topological indices for all the standard graphs is illustrated and the results are discussed. We are planning to find other topological indices for various other standard and chemical graphs with the applications of that in our future work.

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