

An Analysis on Fuzzy Inventory Model without Deficits Using Decagonal Fuzzy Numbers

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Abstract :

This paper proposes a fuzzy economic order quantity and fuzzy optimum total cost for the suggested supply model. Carrying cost and Setup cost are considered in Decagonal fuzzy number. The objective of this study is to select a suitable defuzzification method to obtain the optimum ordering size and effective entire expenditure.

Keywords: Decagonal fuzzy number, fuzzy resource, defuzzification, optimal cost

Introduction:

A fuzzy set is a mathematical representation of vague data that are constructed by natural methods. This model is based on classical set and characteristic function methods. A fuzzy inventory model without shortages using trapezoidal fuzzy number with sensitivity analysis was studied by Dutta and Pavan kumar[3]. The graded

mean integration approach for defuzzification to fuzzy reserve control problem with weibull degradation rate and logarithmic demand rate was studied by Balarama Murthy, et.al [2]. Senbagam and Kavitha Priya [11] constructed an EOQ inventory model with quadratic time dependent demand and two-parameter weibull degradation. Rajalakshmi and Michael Rosario [7] developed a reservoir

model in fuzzy environment under permitted scarcity that are completely backlogged, to estimate the entire price by signed-distance approach . Felix and Victor Devadoss[5] explained the uncertain linguistic environment using decagonal fuzzy number . Senbagam and Kokilamani [10]analysed a fuzzy inventory model which follows Gompertz distributions with shortages and linear demand in which cost is assumed to be heptagonal and octagonal fuzzy numbers.

In particular fuzzy inventory problem with no shortages in which the holding costs and setup cost are represented as Decagonal fuzzy number is considered. The ultimate goal of this study is to find the suitable fuzzification method to find the minimum optimal value. To demonstrate the impact of changes in the parameters of the optimum solution, a numerical example and sensitivity analysis are used. To show the influence of changes in the parameters of the optimum solution, a numerical example and sensitivity analysis are used.

2. Basic Definitions

Definition 2.1[7]. Let X be a non empty set. Then a fuzzy set $\tilde{A}$ in X is characterized by

a function of the form $\mu_{\tilde{A}}(x): X \rightarrow [0, 1]$ and $\mu_{\tilde{A}}$ is called the membership function and for each $x \in X$, $\mu_{\tilde{A}}(x)$ is the degree of membership of x in the fuzzy set $\tilde{A}$.

In other words, A fuzzy set $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) | x \in X\}$ where $\mu_{\tilde{A}}(x): X \rightarrow [0, 1]$. The characteristic function $\mu_{\tilde{A}}(x)$ has only values 0 (false) and 1 (true). Such sets are crisp sets.

Definition 2.2 [5].

A fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10})$ is represented with membership function $\mu_{\tilde{A}}(x)$ as:

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, & a_1 \leq x \leq a_2, \\ \frac{x-a_2}{a_3-a_2}, & a_2 \leq x \leq a_3 \\ \frac{x-a_3}{a_4-a_3}, & a_3 \leq x \leq a_4 \\ \frac{x-a_4}{a_4-a_3}, & a_4 \leq x \leq a_5 \\ 1, & a_5 \leq x \leq a_6 \\ \frac{a_7-x}{a_7-a_6}, & a_6 \leq x \leq a_7 \\ \frac{a_8-x}{a_8-a_7}, & a_7 \leq x \leq a_8 \\ \frac{a_9-x}{a_9-a_8}, & a_8 \leq x \leq a_9 \\ \frac{a_{10}-x}{a_{10}-a_9}, & a_9 \leq x \leq a_{10} \\ 0, & \text{otherwise.} \end{cases}$$

The α -cut of $\tilde{A}$ =
 $(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10})$,
 $0 \leq \alpha \leq 1$ is $A(\alpha) = [A_L(\alpha), A_R(\alpha)]$,

Where $A_{L_1}(\alpha) = a_1 + \alpha(a_2 - a_1) = L_1^{-1}(\alpha)$

$$A_{L_2}(\alpha) = a_2 + \alpha(a_3 - a_2) = L_2^{-1}(\alpha)$$

$$A_{L_3}(\alpha) = a_3 + \alpha(a_4 - a_3) = L_3^{-1}(\alpha)$$

$$A_{L_4}(\alpha) = a_4 + \alpha(a_5 - a_4) = L_4^{-1}(\alpha)$$

and $A_{R_1}(\alpha) = a_7 - \alpha(a_7 - a_6) = R_1^{-1}(\alpha)$

$$A_{R_2}(\alpha) = a_8 - \alpha(a_8 - a_7) = R_2^{-1}(\alpha)$$

$$A_{R_3}(\alpha) = a_9 - \alpha(a_9 - a_8) = R_3^{-1}(\alpha)$$

$$A_{R_4}(\alpha) = a_{10} - \alpha(a_{10} - a_9) = R_4^{-1}(\alpha)$$

Hence

$$L^{-1}(\alpha) = \frac{L_1^{-1}(\alpha) + L_2^{-1}(\alpha) + L_3^{-1}(\alpha) + L_4^{-1}(\alpha)}{4} = \frac{a_1 + a_2 + a_3 + a_4 + \alpha(a_5 - a_1)}{4}$$

$$R^{-1}(\alpha) = \frac{R_1^{-1}(\alpha) + R_2^{-1}(\alpha) + R_3^{-1}(\alpha) + R_4^{-1}(\alpha)}{4} = \frac{a_7 + a_8 + a_9 + a_{10} + \alpha(a_6 - a_{10})}{4}$$

If $\tilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10})$ are decagonal fuzzy number then the signed distance method of $\tilde{A}$ is defined as

$$d(\tilde{A}, \tilde{0}) = \frac{1}{2} \int_0^1 d[A_L(\alpha) + A_R(\alpha)] d\alpha = \frac{a_1 + 2a_2 + 2a_3 + 2a_4 + a_5 + a_6 + 2a_7 + 2a_8 + 2a_9 + a_{10}}{16}$$

If $\tilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10})$ are decagonal fuzzy number then the Graded Mean Integration Value (GMIV) of $\tilde{A}$ is defined as

$$GMIV(\tilde{A}) = \frac{\frac{1}{2} \int_0^1 h(\frac{L^{-1}(h) + R^{-1}(h)}{4}) dh}{\int_0^1 h dh} = \frac{a_1 + 3a_2 + 3a_3 + 3a_4 + 2a_5 + 2a_6 + 3a_7 + 3a_8 + 3a_9 + a_{10}}{24}$$

Symbols and Assumptions:

Symbols:

The following symbols are defined:

D: Total demand over the planning period

H :denotes the fuzzy opportunity cost

S: per-order fuzzy setup cost

T: the period's length

Q: what is the order quantity per cycle?

Assumptions:

The total demand remains static.

The schedule is consistent.

The terms "holding cost" and "ordering cost" are vague.

Shortages are not permitted.

Fuzzy arithmetic model:

In this model, fuzzy holding cost and fuzzy ordering cost are expressed in terms of decagonal fuzzy numbers.

(i) Signed distance method:

Now defuzzifying the total cost

$$\begin{aligned}\widetilde{TC} &= \frac{HT}{2}Q + \frac{DS}{Q} \\ &= \frac{((h_1, h_2, h_3, h_4, h_5, h_6, h_7, h_8, h_9, h_{10}) \otimes T \otimes Q)}{2} \oplus \\ &\quad \frac{((s_1, s_2, s_3, s_4, s_5, s_6, s_7, s_8, s_9, s_{10}) \otimes D)}{Q}\end{aligned}$$

On simplification we get

$$\begin{aligned}\widetilde{TC} &= \\ &\quad \frac{(h_1 + 2h_2 + 2h_3 + 2h_4 + h_5 + h_6 + 2h_7 + 2h_8 + 2h_9 + h_{10}) \otimes T \otimes Q}{32} \\ &\quad \oplus \\ &\quad \frac{(s_1 + 2s_2 + 2s_3 + 2s_4 + s_5 + s_6 + 2s_7 + 2s_8 + 2s_9 + s_{10}) \otimes D}{16Q}\end{aligned}$$

$$\widetilde{TC} = f(Q)$$

The fuzzy optimal order quantity which minimizes the overall stock cost is found by

$$\frac{d}{dQ}(\widetilde{TC}) = 0$$

$$\text{Therefore } Q^* = \sqrt{\frac{2D(s_1 + 2s_2 + 2s_3 + 2s_4 + s_5 + s_6 + 2s_7 + 2s_8 + 2s_9 + s_{10})}{T(h_1 + 2h_2 + 2h_3 + 2h_4 + h_5 + h_6 + 2h_7 + 2h_8 + 2h_9 + h_{10})}}$$

Also $Q = Q^*$ we have $\frac{d^2 f(Q)}{dQ^2} > 0$ which shows that $f(Q)$ is minimum.

Numerical Example:

Let us assume that $H = (1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5, 5, 5.5)$, $S = (10, 20, 30, 40, 50, 60, 70, 80, 90, 100)$, $T = 5$, $D = 50$

We get $Q = 0.0666$, $TC = 191.2750$

(ii) Graded mean integration method:

Now defuzzifying the entire cost

$$\widetilde{TC} = \frac{HT}{2}Q + \frac{DS}{Q}$$

On simplification we get

$$\begin{aligned}\widetilde{TC} &= \\ &\quad \frac{(h_1 + 3h_2 + 3h_3 + 3h_4 + 2h_5 + 2h_6 + 3h_7 + 3h_8 + 3h_9 + h_{10}) \otimes T \otimes Q}{48} \\ &\quad \oplus \\ &\quad \frac{(s_1 + 3s_2 + 3s_3 + 3s_4 + 2s_5 + 2s_6 + 3s_7 + 3s_8 + 3s_9 + s_{10}) \otimes D}{24Q}\end{aligned}$$

$$\widetilde{TC} = f(Q)$$

The fuzzy optimal order quantity which minimizes the total inventory cost is found by

$$\frac{d}{dQ}(\widetilde{TC}) = 0$$

$$\text{Therefore } Q^* = \sqrt{\frac{2D(s_1 + 3s_2 + 3s_3 + 3s_4 + 2s_5 + 2s_6 + 3s_7 + 3s_8 + 3s_9 + s_{10})}{T(h_1 + 3h_2 + 3h_3 + 3h_4 + 2h_5 + 2h_6 + 3h_7 + 3h_8 + 3h_9 + h_{10})}}$$

Sensitivity Analysis:

The consequence of variation in parameters are interpreted as

$\tilde{H}$	Signed distance method		Graded mean integration	
	Q^*	Total Cost	Q^*	Total Cost
(1.5,2,2.5,3,3.5,4,4.5,5,5.5,6)	0.0715	206.0000	0.0584	169.9750
(2,2.5,3,3.5,4,4.5,5,5.5,6,6.5)	0.0761	219.9000	0.0622	181.6750
(2.5,3,3.5,4,4.5,5,5.5,6,6.5,7)	0.0805	233.2500	0.0657	192.5500
(3,3.5,4,4.5,5,5.5,6,6.5,7,7.5)	0.0846	245.7750	0.0691	203.1500

In the above observations, as the values of $\tilde{H}$ increases the optimum vales of Q^* and total cost increases.

Conclusion : We constructed a fuzzy inventory model without shortages in this study, which defuzzifies the holding and setup costs. The expenses are expressed as fuzzy decagonal numbers. When comparing the cost of a fuzzy model using two distinct methods, the graded mean integration

Also $Q = Q^*$ we have $\frac{d^2 f(Q)}{dQ^2} > 0$ which shows that $f(Q)$ is minimum.

Numerical example:

Let us assume that $\tilde{H} = (1,1.5,2,2.5,3,3.5,4,4.5,5,5.5), \tilde{J} = (10,20,30,40,50,60,70,80,90,100), T=5, D=50$

We get $Q = 0.0544, TC = 157.7250$

method outperforms the signed distance method.

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