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An Approach to Solve the Assignment Problem Using Pentagonal Fuzzy Number

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Abstract: Assignment problems were widely applied in the field of production planning ,telecommunication VLSI design ,economic etc. Generally Fuzzy Numbers plays a vital role in many decision making problems. Here an optimal solution of the assignment problem had been derived from Hungarian method .The parameters were being given by using pentagonal fuzzy numbers which are further decomposed into triangular fuzzy numbers. The fuzzy numbers were defuzzfied into real numbers by using pascal's graded mean technique. Numerical example had been given to illustrate the solution procedure.

Keywords: Assignment problem, Hungarian Method ,Pentagonal fuzzy numbers, Triangular fuzzy numbers, Optimization.

Introduction: An Assignment problem is one of the fundamental combinatorial problem in the branch of operation research. The main objective of the assignment problem is to assign number of resources to an equal number of activities in order to minimize the total cost (or) to maximize the profit. Assignment Problem is widely applied in manufacturing sectors and many service systems.Many algorithms had been formulated to find the optimal solution .One of the best known algorithm is Hungarian Algorithm.Suzane Raj, Josephine Vinnarasi,TherasalJeyaseeli [2] solved assignment problem by using pentagonal fuzzy number. D. Selvi ,R. Queen Mary , [5] obtained the optimal solution of the assignment problem using centroid ranking technique.M.Kalaivani, K. Ponnalagu [1] studied the application of pentagonal fuzzy number in solving the Game theory problems.R. Srinivasan , N. Karthikeyan ,A. Jayaraja[3] proposed a ranking method for pentagonal fuzzy number in solving transportationproblem.Shweta Singh ,G.C. Dubey, Rajesh Shrivastava [4] made a Comparative Analysis of Assignment Problem Here an assignment problem had been solved by Hungarian method using

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Pentagonal Fuzzy Number, further it had been decomposed into triangular fuzzy number and analysed the results obtained.

Preliminaries

Fuzzy Set

Let X be a universal set .The Fuzzy Set $\tilde{A}$is defined by the membership function $\mu_{\tilde{A}}(x)$ that maps the element of domain space into unit interval [0,1] where $\mu_{\tilde{A}}$ is the degree of membership of $x \in X$ in the set $\tilde{A}$ and is denoted by $\tilde{A} = \{x, \mu_{\tilde{A}}(x): x \in X\}$.

Fuzzy Number

The Fuzzy Number $\tilde{A}$ is a fuzzy set whose membership function $\mu_{\tilde{A}}(x)$ satisfies the following conditions

(i) $\mu_{\tilde{A}}(x)$ is piecewise continuous
(ii) $\tilde{A}$ is normal if there exist $x \in X$ such that $\mu_{\tilde{A}}(x) = 1$
(iii) $\tilde{A}$ is convex iff $\mu_{\tilde{A}}(\delta x_1 + (1-\delta)x_2) \geq \min\{\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\}$where $\delta \in [0,1]$

Triangular Fuzzy Number

The Fuzzy Number $\tilde{A}^{TrFN} = (a_1, a_2, a_3)$ is a Triangular Fuzzy Number if its membership function is of the form

$$\mu_{\tilde{A}^{TrFN}}(x) = \begin{cases} 0 & x < a_1 \\ \dfrac{x-a_1}{a_2-a_1} & a_1 < x < a_2 \\ 1 & x = a_2 \\ \dfrac{a_3-x}{a_3-a_2} & a_2 < x < a_3 \end{cases}$$

Where $a_1 < a_2 < a_3$

Ranking ofTriangular Fuzzy Number

Let $\tilde{A}^{TrFN} = (a_1, a_2, a_3)$ be a Triangular Fuzzy Number ,by taking the co – efficient of fuzzy number from pascal's triangles and applying simple probability approach the following ranking formula were obtained

$$\mathrm{R}(\tilde{A}^{TrFN}) = \frac{a_1+2a_2+a_3}{4}$$

Arithmetic Operations:

Let $\tilde{A}^{TrFN} = (a_1, a_2, a_3)$ and Number $\tilde{B}^{TrFN} = (b_1, b_2, b_3)$ be any two triangular fuzzy number then

$\tilde{A}^{TrFN} + \tilde{B}^{TrFN} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$,

$\tilde{A}^{TrFN} - \tilde{B}^{TrFN} = (a_1 - b_1, a_2 - b_2, a_3 - b_3)$

Trapezoidal Fuzzy Number

The Fuzzy Number $\tilde{A}^{TraFN} = (a_1, a_2, a_3, a_4)$ is a Trapezoidal Fuzzy Number if its membership function is of the form

$$\mu_{\tilde{A}^{TrFN}}(x) = \begin{cases} 0 & x < a_1 \\ \dfrac{x-a_1}{a_2-a_1} & a_1 < x < a_2 \\ 1 & a_2 < x < a_3 \\ \dfrac{a_4-x}{a_4-a_3} & a_3 < x < a_4 \end{cases}$$

Where $a_1 < a_2 < a_3 < a_4$

Arithmetic Operations:

Let $\tilde{A}^{TraFN} = (a_1, a_2, a_3, a_4)$ and $\tilde{B}^{TraFN} = (b_1, b_2, b_3, b_4)$ be any two trapezoidal fuzzy number then

$\tilde{A}^{TraFN} + \tilde{B}^{TraFN} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$,

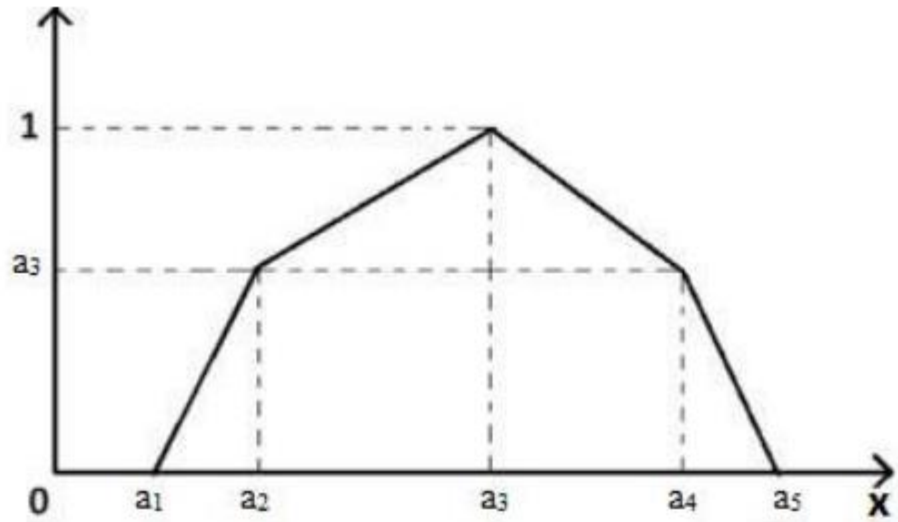
$\tilde{A}^{TraFN} - \tilde{B}^{TraFN} = (a_1 - b_1, a_2 - b_2, a_3 - b_3, a_4 - b_4)$

Pentagonal Fuzzy Number

The Fuzzy Number $\tilde{A}^{PFN} = (a_1, a_2, a_3, a_4, a_5)$ is a Pentagonal Fuzzy Number if its membership function is of the form

$$\mu_{\tilde{A}^{PFN}}(x) = \begin{cases} 0 & x < a_1 \\ \dfrac{x-a_1}{a_2-a_1} & a_1 < x < a_2 \\ \dfrac{x-a_2}{a_3-a_2} & a_2 < x < a_3 \\ 1 & x = a_3 \\ \dfrac{a_4-x}{a_4-a_3} & a_3 < x < a_4 \\ \dfrac{a_5-x}{a_5-a_4} & a_4 < x < a_5 \\ 0 & x > a_5 \end{cases}$$

Where $a_1 < a_2 < a_3 < a_4 < a_5$



Remark :

In the pentagonal fuzzy number $(a_1, a_2, a_3, a_4, a_4)$ if $\frac{a_2+a_3+a_4}{3} = a$ then it can be reduced into triangular fuzzy number as (a_1, a, a_5).

Ranking ofPentagonal Fuzzy Number

Let $\tilde{A}^{PFN} = (a_1, a_2, a_3, a_4, a_5)$ be a Pentagonal Fuzzy Number ,by taking the co – efficient of fuzzy number from pascal's triangles and applying simple probability approach the following ranking formula were obtained

$$R(\tilde{A}^{PFN}) = \frac{a_1+4a_2+6a_3+4a_4+a_5}{16}$$

Arithmetic Operations:

Let $\tilde{A}^{PFN} = (a_1, a_2, a_3, a_4, a_5)$ and $\tilde{B}^{PFN} = (b_1, b_2, b_3, b_4, b_5)$ be any two Pentagonal fuzzy number then

$\tilde{A}^{PFN} + \tilde{B}^{PFN} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4, a_5 + b_5)$,

$\tilde{A}^{PFN} - \tilde{B}^{PFN} = (a_1 - b_1, a_2 - b_2, a_3 - b_3, a_4 - b_4, a_5 - b_5)$

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Practical Situations:

Some of the real time situations werethe idea of assignment had been used is
(i)Assigning Sales man to sales territory
(ii)Assigning vehicles to appropriate routes
(iii)Scheduling teachers to classes etc..

Mathematical Formulation of an Assignment Problem

An Assignment problem can be stated in the form of $n \times n$ cost matrix (c_{ij}) of real numbers in the following table

Jobs / Persons	1	2	3	…..	N
1	c_{11}	c_{12}	c_{13}		c_{1N}
2	c_{21}	c_{22}	c_{23}		c_{2N}
3					
.					
.					
N	c_{N1}	c_{N2}	c_{N3}		c_{NN}

Mathematically assignment problem can be stated as

$$Minimize\ Z = \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}x_{ij}$$

Subject to

$$\sum_{i=1}^{n} x_{ij} = 1 \quad i = 1,2,\dots.n$$

$$\sum_{j=1}^{n} x_{ij} = 1 \quad j = 1,2,\dots.n$$

Where x_{ij} is the decision variable denoting the assignment of the person i to job j ,$c_{ij}$is the cost of assignment of j^{th} job to the i^{th} person .The objective is to minimize the total cost of assigning all the jobs to the available persons

Algorithm:

Step 1: Examine whether the number of rows are equal to the number of columns in the given assignment problem.If not then a dummy row or dummy column must be added with zero cost element.

Step 2:Identify the least cost in each row of the cost matrix ,subtract this least element from each element in that row .Then there will be atleast one zero in each row which is termed as row reduction matrix.

Step 3:In the row reduction matrix identify the least element in each column and subtract from the remaining elements of the corresponding column. Which is termed as column reduction matrix.

Step 4: Examine the row successively until a row with exactly one zeros is found. Encircle the zero element as an assigned cell and cross out all other zero in its column. If there are more than one zero ,then pass onto the next row. The same procedure is followed until all the rows have been examined.

JOURNAL OF ALGEBRAIC STATISTICS
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Step 5: Repeat the same procedure for all the columns of the reduced cost matrix. If there is no single zero in any row or column of the reduced matrix ,then arbitrarily choose a row or column having the minimum number of zeroes. Arbitrarily ,choose zero in the row or column and cross the remaining zeroes in that row or column.

Repeat the steps 4 and 5 until all zeroes are either assigned or crossed out.

Step 6: An optimal assignment were obtained by using the above steps. Otherwise go to the remaining steps.

Step 7: Draw the minimumnumber of horizontal or vertical lines through all the zeroes as follows

(i)Mark (✓) to those rows where no assignment has been made

(ii) Mark (✓) to those columns which have zeroes in the marked rows
(iii) Mark (✓) rows which have assignments in marked columns
(iv)The process may be repeated until no more rows or columns to be examined
(v)Draw straight lines through all unmarked rows and marked columns.

Step 8: If the minimum number of lines passing through all the zeroes is equal to the number of rows or columns ,the optimum solution is attained by an arbitrary allocation in the positions of the zeroes not crossed in step 3.Otherwise we shall move to the next step.

Step 9: Rewrite the cost matrix as follows

(i)Find the elements that are covered by a line .Choose the smallest of these elements and subtract this elements from all the uncrossed elements and add the same at the point of intersection of these lines. Other elements crossed by the lines remain unchanged.

Repeat the step 4 and 5 until an optimum solution were obtained.

Numerical Example

Consider the following assignment problem. Assign four jobs to four machine .The parameters were given in terms of Pentagonal Fuzzy Numbers.

A B
C
D

$$\begin{array}{c} 1\\2\\3\\4 \end{array}\begin{bmatrix} (\mathbf{3},\mathbf{5},\mathbf{7},\mathbf{11},\mathbf{13}) & (\mathbf{2},\mathbf{4},\mathbf{6},\mathbf{8},\mathbf{10}) & (\mathbf{1},\mathbf{3},\mathbf{5},\mathbf{7},\mathbf{9} \\ (\mathbf{2},\mathbf{4},\mathbf{6},\mathbf{8},\mathbf{10}) & (\mathbf{9},\mathbf{11},\mathbf{13},\mathbf{15},\mathbf{17}) & (\mathbf{3},\mathbf{5},\mathbf{7},\mathbf{11},\mathbf{1} \\ (\mathbf{1},\mathbf{3},\mathbf{5},\mathbf{7},\mathbf{9}) & (\mathbf{4},\mathbf{6},\mathbf{8},\mathbf{10},\mathbf{12}) & (\mathbf{0},\mathbf{2},\mathbf{4},\mathbf{6},\mathbf{8} \\ (\mathbf{5},\mathbf{7},\mathbf{9},\mathbf{11},\mathbf{13}) & (\mathbf{11},\mathbf{13},\mathbf{15},\mathbf{17},\mathbf{19}) & (\mathbf{2},\mathbf{4},\mathbf{6},\mathbf{8},\mathbf{10} \end{bmatrix}$$

Solution:

Step 1:

The given assignment problem is balanced, therefore the pentagonal fuzzy cost had been reduced to a crisp number by using Pascal's Graded Technique

$$\begin{array}{c|c} & \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{array}{c} 1\\2\\3\\4 \end{array} & \begin{bmatrix} 7.625 & 6 & 5 & 4 \\ 6 & 13 & 7.625 & 12 \\ 5 & 9 & 4 & 7.625 \\ 9 & 15 & 6 & 4 \end{bmatrix} \end{array}$$

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Step 2:

Row reduced matrix

$$\begin{array}{c} \\ 1\\2\\3\\4 \end{array}\begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 3.625 & 2 & 1 & 0 \\ 0 & 7 & 1.625 & 6 \\ 1 & 4 & 0 & 3.625 \\ 5 & 11 & 2 & 0 \end{bmatrix} \end{array}$$

Step 3:

Column reduced matrix obtained from the above matrix

$$\begin{array}{c} \\ 1\\2\\3\\4 \end{array}\begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 3.625 & 0 & 1 & 0 \\ 0 & 5 & 1.625 & 6 \\ 1 & 2 & 0 & 3.625 \\ 5 & 9 & 2 & 0 \end{bmatrix} \end{array}$$

Step 4: Allocating zeroes for each row and column the resultant assignment were obtained as follows

$$\begin{array}{c} \\ \mathbf{1}\\2\\3\\4 \end{array}\begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 3.625 & (0) & 1 & \mathbf{0} \\ (0) & 5 & 1.625 & \mathbf{6} \\ 1 & 2 & (0) & 3.625 \\ 5 & 9 & 2 & (0) \end{bmatrix} \end{array}$$

Assignment 1→ $\boldsymbol{B}$,2→ $\boldsymbol{A}$, 3 → $\boldsymbol{C}$, 4 → $\boldsymbol{D}$

Total Assignment Value = 20 units

(ii) In the above numerical example if $\frac{a_2+a_3+a_4}{3} = a$ then the pentagonal fuzzy numbers were reduced into triangular fuzzy number as (a_1, a, a_4).

$$\begin{array}{c} \\ 1\\2\\3\\4 \end{array}\begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} (\mathbf{3,7.6,13}) & (\mathbf{2,6,10}) & (\mathbf{1,5,9}) & (\mathbf{0,4,8}) \\ (\mathbf{2,6,10}) & (\mathbf{9,13,17}) & (\mathbf{3,7.6,13}) & (\mathbf{8,12,16}) \\ (\mathbf{1,5,9}) & (\mathbf{4,8,12}) & (\mathbf{0,4,8}) & (\mathbf{3,7.6,13}) \\ (\mathbf{5,9,13}) & (\mathbf{11,15,19}) & (\mathbf{2,6,10}) & (\mathbf{0,4,8}) \end{bmatrix} \end{array}$$

Solution:

Step 1:

The given assignment problem is balanced, therefore the triangular fuzzy cost had been reduced to a crisp number by using Pascal's Graded Technique

$$\begin{array}{c} \\ 1\\2\\3\\4 \end{array}\begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 7.8 & 6 & 5 & 4 \\ 6 & 13 & 7.8 & 12 \\ 5 & 9 & 4 & 7.8 \\ 9 & 15 & 6 & 4 \end{bmatrix} \end{array}$$

Step 2:

Row reduced matrix

$$\begin{array}{c} \\ 1\\2\\3\\4 \end{array}\begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 3.8 & 2 & 1 & 0 \\ 0 & 7 & 1.8 & 6 \\ 1 & 4 & 0 & 3.8 \\ 5 & 11 & 2 & 0 \end{bmatrix} \end{array}$$

Step 3: Column reduced matrix obtained from the above matrix

$$\begin{array}{c} \\ 1\\2\\3\\4 \end{array}\begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 3.8 & 0 & 1 & 0 \\ 0 & 5 & 1.8 & 6 \\ 1 & 2 & 0 & 3.8 \\ 5 & 9 & 2 & 0 \end{bmatrix} \end{array}$$

Step 4: Allocating zeroes for each row and column the resultant assignment were obtained as follows

$$\begin{array}{cc} & \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 3.8 & (0) & 1 & 0 \\ (0) & 5 & 1.8 & 6 \\ 1 & 2 & (0) & 3.8 \\ 5 & 9 & 2 & (0) \end{bmatrix} \end{array}$$

Assignment 1→ $\boldsymbol{B}$,2→ $\boldsymbol{A}$, $\mathbf{3} \rightarrow \boldsymbol{C}$, $\mathbf{4} \rightarrow \boldsymbol{D}$

Total Assignment Value = 20 units

Comparison Table

Fuzzy Numbers	Assignment Value
Pentagonal Fuzzy Numbers	20
Triangular Fuzzy Numbers	20

Conclusion :

In this paper the assignment problem were solved by Hungarian Method by using pentagonal fuzzy number ,further it had been converted into triangular fuzzy number .The results yield for both the fuzzy numbers were same.

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