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Significance of Hmt Effects on Hydromagnetic Oscillatory Jeffrey Fluid Through a Tube

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Abstract

An unsteady electrically conducting, viscous, incompressible, hydromagnetic oscillatory (or periodic) Jeffrey fluid through an insulated vertical porous tube is considered. A magnetic field is applied externally in a uniform manner and perpendicular to the tube. Boussinesq approximation is used to formulate the mathematical equations of Jeffrey fluid. The mathematical equations are solved one by one to attain the exact analytical solution of concentration, temperature and hence velocity profiles. The effects of different parameters like Hartmann Number, Peclet Number, Schmidt Number, Grashoff Number on the flow of Jeffrey fluid are drawn graphically and explained in detail.

Keywords: Jeffrey fluid; porous medium; vertical tube; Hydromagnetic or MHD Magneto Hydro Dynamics; oscillatory; Heat and Mass Transfer (HMT)

1. Introduction

The fluid flow that conducts electricity has applications in production of hydromagnetic generators, study of plasmas, formation of nuclear reactors, and extraction of geothermal energy and so on. Various researchers like Nigam et al. (1960), Soundalgekar et al. (1971), Raptis et al. (1982), Aldoss et al. (1995), Attia et al. (1996), Makinde et al. (2005), Mostafa (2009), Hamza et al. (2011) etc. made a detailed study on hydromagnetic oscillatory fluid through various channels and under various suitable conditions.

Vijayalakshmi et al. (2015) made an in-depth research on the chemical reaction effects on hydromagnetic periodic fluid flowing through a channel containing small minute pores. Govindarajan et al. (2015, 2016, 2018, 2019) have made a detailed discussion on chemical effects and heat transfer on hydromagnetic periodic fluid through different / various kinds of channels.

In nature, Non-Newtonian fluids are more common when compared to Newtonian fluids. Al Khatib et al. (2001) analysed the flow of Poiseuille fluid through a channel. Frigaard et al. (2004) studied the flow of a viscoplastic fluid through a non-uniform channel of slowly varying width. Aamir Ali et al. (2011) analyzed the oscillatory Chanel flow for non-Newtonian of fluid. The flow in the channel is driven by suction at the permeable walls, while oscillations in the velocity field are caused by tiny amplitude time harmonic pressure waves. Rita et al. (2012) observed the impact of heat transfer on hydromagnetic oscillatory viscoelastic fluid through porous media.

Owing to the various applications of Non-Newtonian fluid, the significance of HMT (Heat and Mass Transfer) effects on hydromagnetic flow of Jeffrey fluid through a tube containing small pores is considered in this paper. The analytical expressions for concentration, temperature as well as the velocity of the Jeffrey fluid are calculated. The effects of different dimensionless (non dimensional) parameters (variables) on the concentration, temperature as well as the velocity of Jeffrey fluid are drawn and explained well clearly through graphs.

2. Mathematical Form

The geometry or physical model of Jeffrey fluid through a tube of thickness h and is subjected to the control of electrically applied magnetic field and heat transfer due to radiation is exposed in **Fig.1**. It is supposed to have that the

Jeffrey fluid under consideration possesses less electrical conduction and due to that behaviour, the electromagnetic force generated by the fluid is also very small. Select the coordinates (*x*, *y*), where the direction of *x* is chosen all along the middle of the tube and the direction of *y* axis is taken in a way such that it is perpendicular to the direction of Jeffrey fluid flow.

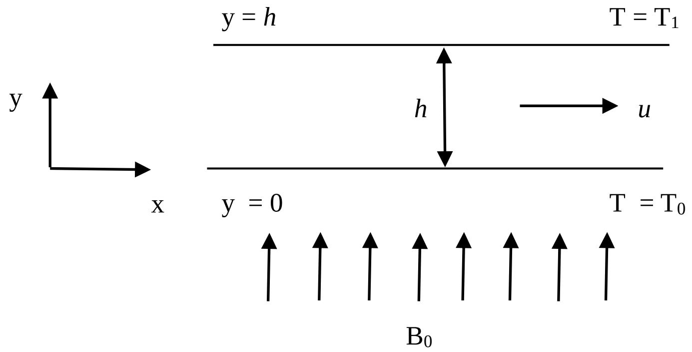

Fig. 1 Physical Model

It is noteworthy to take the constitutive equation S for Jeffrey fluid as

$$S = \frac{\mu}{1+\lambda_1}(\dot{\gamma} + \lambda_2 \ddot{\gamma}) \tag{1}$$

γ rate of shear and dots above γ represent the first and second derivatives with respect to time.

Governing Equations

The fundamental equations of momentum, energy & concentration based on Boussinesq approximations are

$$\rho \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \frac{\mu}{1+\lambda_1}\frac{\partial^2 u}{\partial y^2} - \sigma B_0^2 u + \rho g \beta (T - T_0) + \rho g \beta_c (C - C_o) \tag{2}$$

$$\rho \frac{\partial T}{\partial t} = \frac{k}{c_p}\frac{\partial^2 T}{\partial y^2} - \frac{1}{c_p}\frac{\partial q}{\partial y} \tag{3}$$

$$\rho \frac{\partial C}{\partial t} = D_m \frac{\partial^2 C}{\partial y^2} - K_c (C - C_0) \tag{4}$$

The wall conditions of the tube are selected in such a way that

$u=0,\ T=T_0\ C=C_0$ at the lower end $y=0$ (5)

$u=0,\ T=T_1\ C=C_1$ at the lower end $y=h$ (6)

According to Cogley et al. (1968), the differential approximation for radiative heat transport is provided by

$$\frac{\partial q}{\partial y} = 4\alpha^2 (T_0 - T) \tag{7}$$

Dimensionless variables

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$$\bar{x}=\frac{x}{h},\ \bar{y}=\frac{y}{h},\ \bar{u}=\frac{u}{U},\ \theta=\frac{T-T_0}{T_1-T_0},\ \bar{t}=\frac{tU}{h},\ \bar{p}=\frac{p\,a}{\mu U},\ M^2=\frac{\sigma h^2 B_0^2}{\mu},\ Gr=\frac{\rho g\,\beta(T-T_0)}{U\mu},$$

$$Gc=\frac{\rho g\,\beta_c(C_1-C_0)}{U\mu},\ \mathrm{Re}=\frac{\rho hU}{\mu},\ \ Pe=\frac{\rho hU c_p}{k},\ N^2=\frac{4\alpha^2 h^2}{k},\ \ Sc=\frac{Ua}{Dm},\ \ \overline{K}_c=\frac{K_c\,a}{U\mu}$$

After applying dimensionless parameters, the above equations (2), (3) and (4) are reduced to

$$\mathrm{Re}\frac{\partial u}{\partial t}=-\frac{\partial p}{\partial x}+\frac{1}{1+\lambda_1}\frac{\partial^2 u}{\partial y^2}-M^2u+Gr\theta+Gc\varphi \tag{8}$$

$$Pe\frac{\partial \theta}{\partial t}=\frac{\partial^2\theta}{\partial y^2}+N^2\theta \tag{9}$$

$$\frac{\partial\varphi}{\partial t}=\frac{1}{Sc}\frac{\partial^2\varphi}{\partial y^2}+K_c\varphi \tag{10}$$

The non-dimensional boundary wall conditions of the tube are

$u = 0, \theta = 0, \varphi = 0$ at $y = 0$ (11)

$u = 0, \theta = 1, \varphi = 1$ at $y = 1$ (12)

3. Method of Solution

As Jeffrey fluid under consideration is MHD oscillatory, take

$$-\frac{\partial p}{\partial x}=\lambda e^{i\omega t} \tag{13}$$

$$u(y,t)=u_0(y)e^{i\omega t} \tag{14}$$

$$\theta(y,t)=\theta_0(y)e^{i\omega t} \tag{15}$$

$$\varphi(y,t)=\varphi_0(y)e^{i\omega t} \tag{16}$$

Substitute the above equations in (8), (9) and (10). Then

$$\frac{d^2u_0}{dy^2}-m_1^2u_0=-\lambda(1+\lambda_1)-Gr(1+\lambda_1)\theta_0-Gc(1+\lambda_1)\varphi_0 \tag{17}$$

$$\frac{d^2\theta_0}{dy^2}+m_2^2\theta_0=0 \tag{18}$$

$$\frac{d^2\varphi_0}{dy^2}+m_3^2\varphi_0=0 \tag{19}$$

with boundary wall conditions

$u_0 = 0, \theta_0 = 0, \varphi = 0$ at $y = 0$ (20)

$u_0 = 0, \theta_0 = 1, \varphi = 1$ at $y = 1$ (21)

Here $m_1=\sqrt{(1+\lambda_1)\,(M^2+i\omega\mathrm{Re})}$, $m_2=\sqrt{N^2-i\omega Pe}$, $m_3=\sqrt{(1+\lambda_1)\,(K_c+i\omega)Sc}$

Velocity, Temperature and Concentration

Solve equations (17), (18) and (19) using the boundary conditions (20) and (21).

$$u=\left(t_1\left(\frac{\sinh m_1(y-1)}{\sinh m_1}\right)-(t_1+t_2+t_3)\frac{\sinh m_1 y}{\sinh m_1}+t_1+t_2\frac{\sin m_1 y}{\sin m_1}+t_3\frac{\sinh m_3 y}{\sinh m_3}\right)e^{i\omega t} \tag{22}$$

$$\theta=\left(\frac{\sinh(m_2 y)}{\sinh m_2}\right)e^{i\omega t} \tag{23}$$

$$\varphi=\left(\frac{\sinh(m_3 y)}{\sinh m_3}\right)e^{i\omega t} \tag{24}$$

4. Graphical Results & Discussions

To study the HMT effects on radiative hydromagnetic oscillatory Jeffrey fluid through a vertical tube, the species concentration φ, temperature θ as well as the velocity *u* profiles are depicted through various figures **(Fig. 2 – Fig. 12)** against *y* for different physical parameters. The graphs are drawn using MATLAB R2021a. The dimensionless parameters are taken initially as Re =1, *N* = 1, *Gr* = 1, *Pe* = 0.71, ω = 1, *t* = 0.5 and *M* = 1.

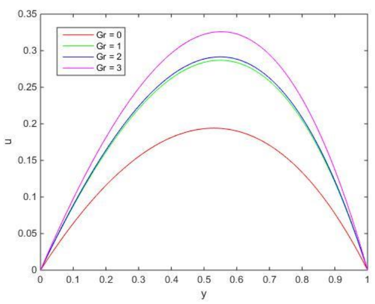

Fig. 2 Impact of *Gr* on *u*

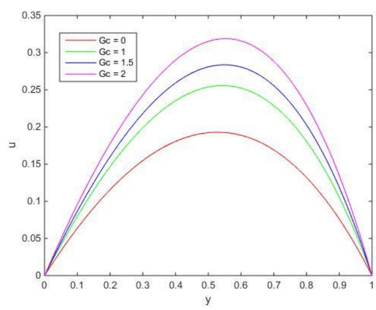

Fig. 3 Impact of *Gc* on *u*

From **Fig. 2** and **Fig. 3**, it is clear that velocity of Jeffrey fluid accelerates with an enhancement of Grashof number Gr due to transfer of heat and Grashof number Gc due to transfer of mass.

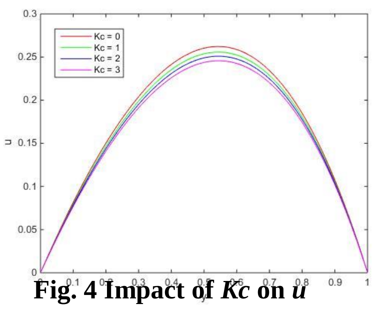

Fig. 4 Impact of *Kc* on *u*

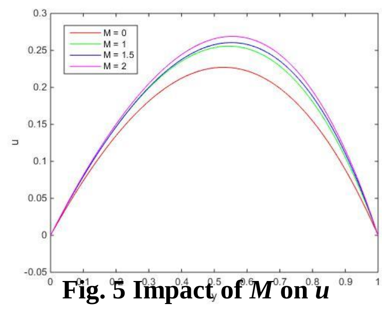

Fig. 5 Impact of *M* on *u*

From **Fig. 4**, it is evident that the velocity of Jeffrey fluid decelerates with an enhancement of Concentration parameter Kc. It is also well understood that the velocity of fluid increases with an acceleration for the values of Hartmann number M **(Fig. 5)**.

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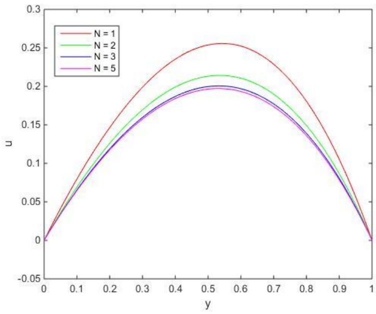

Fig. 6 Impact of *N* on *u*

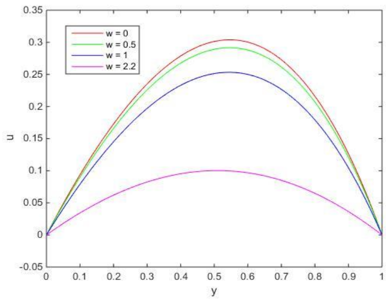

Fig. 7 Impact of ω on *u*

From **Fig. 6** and **Fig. 7**, it is noted that velocity of Jeffrey fluid decelerates with an increment in the values of radiation parameter N and frequency of oscillation ω.

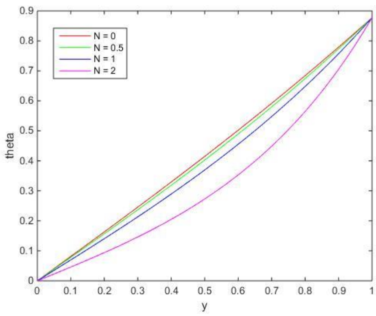

Fig. 8 Impact of N on θ

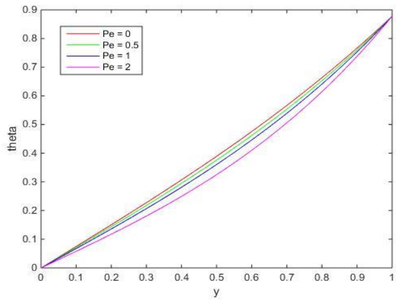

Fig. 9 Impact of Pe on θ

From **Fig. 8** and **Fig. 9**, it is obvious that temperature of Jeffrey fluid decreases with an increment in the value of radiation parameter N and Peclet number Pe.

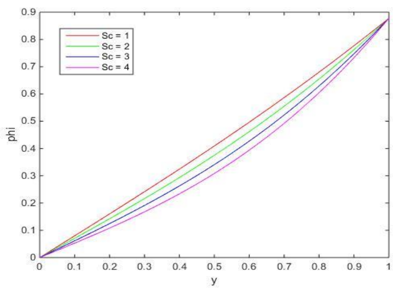

Fig. 10 Impact of Sc on φ

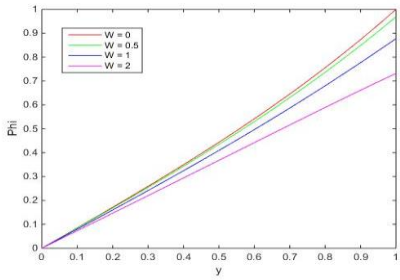

Fig. 11 Impact of ω on φ

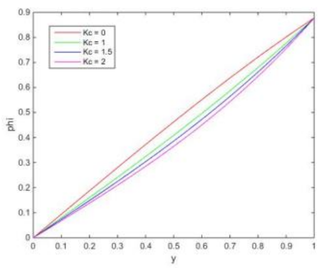

Fig. 12 Impact of Kc on φ

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From the above figures **Fig. 10, Fig. 11** and **Fig. 12**, it is known that concentration of Jeffrey fluid falls with a rise in the value of Schmidt number Sc, frequency of oscillation ω and concentration parameter Kc.

5. Conclusion

In this current paper, the significance of HMT effects on hydromagnetic oscillatory Jeffrey fluid through a vertical tube is studied elaborately. The exact solutions for the concentration, temperature and velocity of Jeffrey fluid are calculated analytically using ordinary differential equations method. It is established that, the velocity *u* of Jeffrey fluid accelerates with increasing Gr, Gc and M, while it decelerates with increasing Kc, N, ω and Pe. The temperature θ of Jeffrey fluid is decreasing with increasing N, Pe and ω. Also, it is well brought to our notice that the concentration φ of Jeffrey fluid decreases with an acceleration in the value of Sc, Kc and ω. As $\lambda_1 \to 0$ and $\lambda_2 \to 0$, the results or outcomes obtained in this paper agree with the results of Vijayalakshmi et al. (2015).

6. References

[1] Aamir Ali, Saleem Asghar, Oscillatory channel flow for Non-Newtonian fluid, *International Journal of Physical Sciences*, 6 (36), (2011), pp. 8035 - 8043.
[2] T. K. Aldoss, M. Alkam, M. Shatarah, Natural convection from a horizontal annulus partially filled with porous medium, *International Communications in Heat and Mass transfer*, 31 (3), (2004), pp. 441 - 452.
[3] M.A.M. Al Khatib, S.D.R. Wilson, The development of Poiseuille flow of a yield-stress fluid, *J. Non-Newtonian Fluid Mech*., 100 (2001), pp. 1–8.
[4] H. A. Attia, and N. A. Kotb, MHD flow between two parallel plates, *Acta Mechanica*, 117 (1996), pp. 215-220.
[5] A.C.L. Cogley, W.G. Vinvent, and E.S. Giles, Differential Approximation for Radiative Heat Transfer in a Non-Grey Gas near Equilibrium, *American Institute of Aeronautics and Astronautics*, 6 (1968), pp. 551-553.
[6] I.A. Frigaard and D.P. Ryan, Flow of a visco-plastic fluid in a channel of slowly varying width, *J. Non-Newtonian Fluid Mech*., 123 (2004), pp. 67–83.
[7] M. M. Hamza B.Y Isah H. Usman, Unsteady heat transfer to MHD oscillatory flow through a porous medium under slip condition, *International Journal of Computer Applications*, 33(4) (2011), pp. 12-17.
[8] O. D. Makinde, and P. Y. Mhone, Heat Transfer to MHD oscillatory flow in a channel filled with porous medium, *Rom. J. Phys*., 50 (2005), pp. 931-938.
[9] A.A. Mostafa, Thermal radiation effect on unsteady MHD free convection flow past a vertical plate with temperature-dependent viscosity, *The Canadian journal of chemical engineering*, 87(1) (2009), pp. 47-52.
[10] S. D. Nigam, and S. N. Singh, Heat transfer by laminar flow between parallel plates under the action of transverse magnetic field, *Q. J. Mech. Appl. Math.*, 13 (1960), pp. 85-87.
[11] Raptis, C. Massias, and G. Tzivanidis, Hydromagnetic free convection flow through a porous medium between two parallel plates, *Phys. Lett*., 90A (1982), pp. 288-289.
[12] Rita and U. Jyoti das, Heat Transfer to MHD oscillatory Viscoelastic flow in a channel filled with porous medium, *Physics Research International*, (2012), pp. 1-5.
[13] V. M. Soundalgekar, and J. P. Bhat, Oscillatory MHD channel flow and heat transfer, *J. Pure Appl. Math*., 15 (1971), pp. 819-828.
[14] R. Vijayalakshmi, A. Govindarajan, J. Sasikumar, E. P. Siva, Chemical Reaction Effects on Radiative MHD Oscillatory Flow in a Porous Channel with Mass Transfer and Heat Absorption, *International Journal of Scientific & Engineering Research*, Vol. 6 (3), (2015), pp. 199 - 203.
[15] M. Vidhya, R. Vijayalakshmi, A. Govindarajan, Chemical Reaction Effects on Radiative MHD Oscillatory Flow in a Porous Channel with Heat and Mass Transfer in an Asymmetric Channel, *ARPN Journal of Engineering And Applied Sciences*, Vol. 10 (4), (2015), pp. 1839 - 1845.
[16] R. Vijayalakshmi, A. Govindarajan, Slip Effect on Radiative MHD Oscillatory Flow with Chemical Reaction in an Asymmetric Channel, *Global Journal of Pure and Applied Mathematics*, Vol. 12 (2), (2016), pp. 43 - 49.
[17] R. Vijayalakshmi, A. Govindarajan, Chemically Reacting MHD Oscillatory Slip Flow in an Asymmetric Channel with both Concentration and Temperature Variation, *International Journal of Pure and Applied Mathematics*, Vol. 114 (6), (2017), pp. 61 - 66.

[18] Govindarajan, R. Vijayalakshmi, V. Ramamurthy, Combined Effects of Heat and Mass Transfer to MHD Oscillatory Dusty Fluid Flow in a Porous Channel, *Journal of Physics: Conference Series*, 1000, (2018), pp. 1 - 11.

[19] Govindarajan, R. Vijayalakshmi, Effect of Hall Current in MHD Oscillatory Couple Stress Dusty Fluid through an Inclined Saturated Permeable Channel, *International Journal of Engineering & Technology*, Vol. 7, (2018), pp. 801 - 805.

[20] R. Vijayalakshmi, A. Govindarajan, Effect of Partial Slip on Hydromagetic Oscillatory Dusty Fluid Flow in an AsymmetricWavy Channel, *AIP Conference Proceedings*, 2112, (2019), pp. 1 - 9.

[21] A. Govindarajan, R. Vijayalakshmi, Energy Transfer Effects in Hydromagetic Oscillatory Dusty Fluid Flow in a Vertical Porous Channel, *AIP Conference Proceedings*, 2112, (2019), pp. 1 - 8.

[22] R. Vijayalakshmi, A. Govindarajan, Impact of Heat Transfer and Viscous Dissipation on Unsteady MHD oscillatory Dusty Fluid Flow Through a Vertical Porous Channel, *Journal of Physics: Conference Series*, 1377, (2019), pp. 1 - 16.

7. Appendix Nomenclature

B_0	Electromagnetic induction
C	Concentration of the fluid
C_p	Specific heat at constant pressure
D_m	Mass diffusion coefficient
g	Gravitational force
Gr	Grashof number due to the transfer of heat
Gc	Grashof number due to the transfer of mass
M	Hartmann number
k	Thermal conductivity
Kc	Chemical reaction parameter
N	Radiation parameter
p	Pressure
Pe	Peclet number
q	Radiative heat flux
Re	Reynolds number
Sc	Schmidt number
t	Time variable
T	Fluid Temperature
T_0	Temperature at y = 0
T_1	Temperature at y = *h*
u	Axial velocity
u	Mean flow velocity
x	Axial distance
y	Transverse distance
μ	Dynamic viscosity
α	Mean radiation absorption coefficient
β	Volume expansion coefficient due to temperature
βc	Volume expansion coefficient due to concentration
λ	Real constant
σ	Conductivity of the fluid
ω	Frequency of the oscillation
ρ	Fluid density
θ	Temperature
λ_1	Ratio of relaxation time to retardation time
λ_2	Retardation time