JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 3265-3271
https://publishoa.com
ISSN: 1309-3452

Geometric Mean Cordial Labeling of Helm Graph

V Annamma[1], Jawahar Nisha M I[2]

[1]Assistant Professor of Mathematics, L.N. Government College, Ponneri

[2]Assistant Professor of Mathematics, J.B.A.S College for Women, Teynampet, Chennai

Email: nishabdul@gmail.com

Received 2022 March 25; **Revised** 2022 April 28; **Accepted** 2022 May 15.

ABSTRACT

Let G be a graph with vertex set $V(G)$ and edge set $E(G)$. Let $f: V(G) \rightarrow \{0, 1, 2\}$ be a mapping. Assign the label $\left\lceil\sqrt{f(u)f(v)}\right\rceil$ for each edge $uv. f$ is called a *geometric mean cordial (GMC) labeling* if $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$, where the number of vertices and edges labeled with x for $x \in \{0, 1, 2\}$ is denoted by $v_f(x)$ and $e_f(x)$ respectively. A graph with a *GMC labeling* is called a *GMC graph*. In this research article we investigate the *GMC labeling* of Helm graph H_n and also establish the GMC labeling of some graph operations on Helm graph such as Fusion and Duplication. We also prove that the graph operation Switching on Helm graph does not admit GMC labeling.

Keywords: Duplication, Fusion, Geometric Mean Cordial Labeling, Helm graph, Switching

AMS Subject Classification: 05C78

INTRODUCTION

In this research article we consider only simple, finite, undirected and connected graphs. An assignment of numbers which are integers to the edges or vertices, or both with some conditions is called graph labeling. Let G be a graph. Let $V(G)$ denote vertex set of G and $E(G)$ denote edge set of G. Let $f: V(G) \rightarrow \{0, 1, 2\}$ be a mapping. Define the induced edge labeling $f^*: E(G) \rightarrow \{0,1,2\}$ by $\left\lceil\sqrt{f(u)f(v)}\right\rceil$ $\forall$ edge $uv. f$ is called a geometric mean cordial (GMC) labeling if $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$, for all $0 \leq i, j \leq 2$. Here $v_f(x)$ represents the number of vertices labeled with x and $e_f(x)$ represents the number of edges labeled with x where $x \in \{0, 1, 2\}$. A graph that admits GMC labeling is a GMC graph [4]. In this research article we discuss the GMC labeling of the graph Helm H_n and also establish the GMC labeling of few graph operations on Helm graph such as Fusion and Duplication. We also investigate that the graph operation Switching on Helm graph does not admit GMC labeling.

PRELIMINARIES

Definition 2.1. [5]

The wheel W_n is the graph obtained by combining K_1 and C_n i.e. $W_n = K_1 + C_n$. The apex of the wheel W_n is the vertex corresponding to K_1. The vertices on the rim of the wheel W_n are the vertices of C_n and the edges on the rim of the wheel W_n are the edges of C_n.

Definition 2.2. [3]

The Helm graph H_n is constructed by joining a pendant edge to each vertex on the rim of a wheel W_n.

Definition 2.3. [4]

The set of all vertices which are adjacent to the vertex v in G is called open neighbourhood of the vertex v in G.

Definition 2.4. [2]

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ISSN: 1309-3452

The fusion of any two vertices u and v of G as a single vertex w, gives a new graph G' in such a way that the edges incident with either u or v in G is now incident with w in G'.

Definition 2.5. [4]

The duplication of any vertex u of G by another new vertex w gives a graph G' such that the neighbourhood of w is the neighbourhood of u.

Definition 2.6. [2]

A switching of any vertex v in a graph G is constructed by removing all the edges which are incident with v and joining the vertex v with the vertices v_i, where the vertices v_i are not adjacent to v in G.

RESULTS

Theorem 3.1: The graph helm H_n is a GMC graph if $n \geq 5$.

Proof: Let Helm H_n be a graph whose vertex set $V(H_n) = \{u, u_i, v_i: 1 \leq i \leq n\}$ and its edge set $E(H_n) = \{uu_i, u_iv_i : 1 \leq i \leq n\} \cup \{u_iu_{i+1}: 1 \leq i \leq n-1\} \cup \{u_nu_1\}$. Let l denote the number of vertices and k denote the number of edges. Then $l = 2n+1$ and $k = 3n$.

Define $f : V(G) \to \{0, 1, 2\}$ as shown below:

Case (i): $\boldsymbol{n \equiv 0 (mod\ 3)}$

Let $n = 3t, t > 1$

$$f(u) = 1$$

$$f(u_i) = \begin{cases} 0, & 1 \leq i \leq t-1 \\ 2, & t \leq i \leq 2t-1 \\ 1, & 2t \leq i \leq 3t \end{cases} \quad \text{and} \quad f(v_i) = \begin{cases} 0, & 1 \leq i \leq t+1 \\ 2, & t+2 \leq i \leq 2t+1 \\ 1, & 2t+2 \leq i \leq 3t \end{cases}$$

Then $v_f(0) = v_f(2) = 2t, v_f(1) = 2t+1$ and $e_f(0) = e_f(1) = e_f(2) = 3t$.

Case (ii): $\boldsymbol{n \equiv 1 (mod\ 3)}$

Let $n = 3t + 1, t > 1$

$$f(u) = 1$$

$$f(u_i) = \begin{cases} 0, & 1 \leq i \leq t-1 \\ 2, & t \leq i \leq 2t-1 \\ 1, & 2t \leq i \leq 3t+1 \end{cases} \quad \text{and} \quad f(v_i) = \begin{cases} 0, & 1 \leq i \leq t+2 \\ 2, & t+3 \leq i \leq 2t+3 \\ 1, & 2t+4 \leq i \leq 3t+1 \end{cases}$$

Then $v_f(0) = v_f(1) = v_f(2) = 2t+1$ and $e_f(0) = e_f(1) = e_f(2) = 3t+1$.

Case (iii): $\boldsymbol{n \equiv 2 (mod\ 3)}$

Let $n = 3t + 2, t \geq 1$

$$f(u) = 1$$

$$f(u_i) = \begin{cases} 0, & 1 \leq i \leq t \\ 2, & t+1 \leq i \leq 2t \\ 1, & 2t+1 \leq i \leq 3t+2 \end{cases} \quad \text{and} \quad f(v_i) = \begin{cases} 0, & 1 \leq i \leq t+1 \\ 2, & t+2 \leq i \leq 2t+3 \\ 1, & 2t+4 \leq i \leq 3t+2 \end{cases}$$

Then $v_f(0) = 2t+1, v_f(1) = v_f(2) = 2t+2$ and $e_f(0) = e_f(1) = e_f(2) = 3t+2$.

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ISSN: 1309-3452

From all the above three cases, we see that $\left|v_f(i) - v_f(j)\right| \leq 1$ and $\left|e_f(i) - e_f(j)\right| \leq 1$ for all $i, j \in \{0, 1, 2\}$.

Hence f is a GMC labeling.

Illustration 3.1: GMC Labeling of graph Helm H_{52}

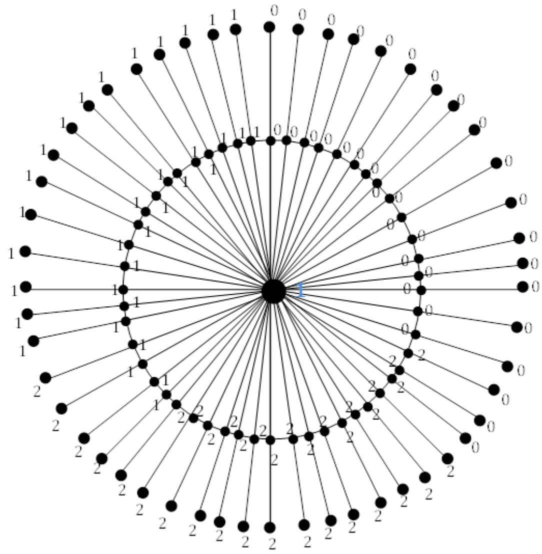

Figure 1. GMC Labeling of Helm H_{52}.

Here $v_f(0) = v_f(2) = v_f(1) = 35$ and $e_f(0) = e_f(1) = e_f(2) = 52$.

Therefore $\left|v_f(i) - v_f(j)\right| \leq 1$ and $\left|e_f(i) - e_f(j)\right| \leq 1$ for all $i, j \in \{0, 1, 2\}$.

Note: H_3 and H_4 does not admit GMC labeling.

Theorem 3.2: The graph formed by fusing any two pendent vertices of Helm graph H_n is a GMC graph if $n \geq 5$ and $n \neq 6$.

Proof: Let G_f represents the graph formed by fusing any two pendent vertices v_i and v_j as one vertex u' in a Helm graph H_n. Then its vertex set $V(G_f) = \{u, u', u_i, v_j : 1 \leq i \leq n, 1 \leq j \leq n-2\}$ and its edge set $E(G_f) = \{uu_i, u_jv_j : 1 \leq i \leq n, 1 \leq j \leq n-2\} \cup \{u_{n-1}u', u_nu'\} \cup \{u_iu_{i+1} : 1 \leq i \leq n-1\} \cup \{u_nu_1\}$.

Let $\left|V(G_f)\right| = l$ and $\left|E(G_f)\right| = k$. Then $l = 2n, k = 3n$.

Define $f : V(G) \to \{0, 1, 2\}$ as shown below:

Case (i): $\boldsymbol{n \equiv 0 (mod\ 3)}$

Let $n = 3t, t > 2$

$$f(u) = 1, f(u') = 1$$

$$f(u_i) = \begin{cases} 0, & 1 \leq i \leq t-1 \\ 2, & t \leq i \leq 2t-1 \\ 1, & 2t \leq i \leq 3t \end{cases} \quad \text{and} \quad f(v_i) = \begin{cases} 0, & 1 \leq i \leq t+1 \\ 2, & t+2 \leq i \leq 2t+1 \\ 1, & 2t+2 \leq i \leq 3t-2 \end{cases}$$

Then $v_f(0) = v_f(1) = v_f(2) = 2t$ and $e_f(0) = e_f(1) = e_f(2) = 3t$.

Case (ii): $\boldsymbol{n \equiv 1 (mod\ 3)}$

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https://publishoa.com
ISSN: 1309-3452

Let $n = 3t + 1, t > 1$

$$f(u) = 1, f(u') = 2$$

$$f(u_i) = \begin{cases} 0, & 1 \le i \le t-1 \\ 2, & t \le i \le 2t-1 \\ 1, & 2t \le i \le 3t+1 \end{cases} \quad \text{and} \quad f(v_i) = \begin{cases} 0, & 1 \le i \le t+2 \\ 2, & t+3 \le i \le 2t+1 \\ 1, & 2t+2 \le i \le 3t-1 \end{cases}$$

Then $v_f(0) = 2t, v_f(1) = v_f(2) = 2t+1$ and $e_f(0) = e_f(1) = 3t+1, e_f(2) = 3t$.

Case (iii): $n \equiv 2(mod\ 3)$

Let $n = 3t + 2, t \ge 1$

$$f(u) = 1, f(u') = 2$$

$$f(u_i) = \begin{cases} 0, & 1 \le i \le t \\ 2, & t+1 \le i \le 2t \\ 1, & 2t+1 \le i \le 3t+2 \end{cases} \quad \text{and} \quad f(v_i) = \begin{cases} 0, & 1 \le i \le t+1 \\ 2, & t+2 \le i \le 2t+1 \\ 1, & 2t+2 \le i \le 3t \end{cases}$$

Then $v_f(0) = v_f(2) = 2t+1, v_f(1) = 2t+2$ and $e_f(0) = e_f(1) = e_f(2) = 3t+2$.

From all the above three cases, we see that $|v_f(i) - v_f(j)| \le 1$ and $|e_f(i) - e_f(j)| \le 1$ for all $i, j \in \{0, 1, 2\}$.

Hence f is a GMC labeling.

Illustration 3.2: Fusion of vertices v_{29} and v_{30} of H_{30} and its GMC labeling.

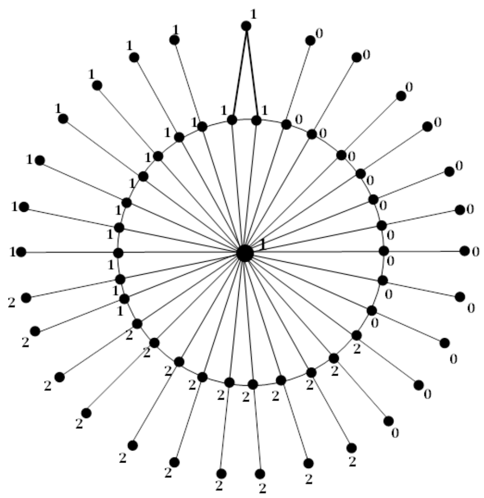

Figure 2. Fusion of vertices v_{29} and v_{30} of H_{30} and its GMC labeling.

Here $v_f(0) = v_f(2) = v_f(1) = 20$ and $e_f(0) = e_f(1) = e_f(2) = 30$.

Therefore $|v_f(i) - v_f(j)| \le 1$ and $|e_f(i) - e_f(j)| \le 1$ for all $i, j \in \{0, 1, 2\}$.

Theorem 3.3: The graph formed by duplication of any arbitrary pendent vertex v_i of graph Helm H_n is a GMC graph if $n \ge 5$.

Proof: Let G_d represents the graph formed by duplication of any arbitrary pendent vertex v_i of a graph Helm H_n. Then its vertex set $V(G_d) = \{u, u_i, v_i: 1 \le i \le n\} \cup \{v_i': i = 1 \text{ or } 2 \text{ or} \dots n\}$ and its edge set $E(G_d) = \{uu_i, u_iv_i : 1 \le i \le n\} \cup \{u_iv_i': i = 1 \text{ or } 2 \text{ or} \dots n\} \cup \{u_iu_{i+1}: 1 \le i \le n-1\} \cup \{u_nu_1\}$. Let $|V(G_f)| = l$ and $|E(G_f)| = k$. Then $l = 2n +$

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https://publishoa.com
ISSN: 1309-3452

$2, k = 3n + 1$.

Define $f : V(G) \to \{0, 1, 2\}$ as shown below:

Case (i): $\boldsymbol{n \equiv 0 (mod\ 3)}$

Let $n = 3t, t > 1$

$f(u) = 1, f(v_i') = 0$ for $i = 1$ or 2 or … n

$$f(u_i) = \begin{cases} 0, & 1 \le i \le t-1 \\ 2, & t \le i \le 2t-1 \\ 1, & 2t \le i \le 3t \end{cases} \quad \text{and } f(v_i) = \begin{cases} 0, & 1 \le i \le t+1 \\ 2, & t+2 \le i \le 2t+1 \\ 1, & 2t+2 \le i \le 3t \end{cases}$$

Then $v_f(0) = v_f(1) = 2t + 1, v_f(2) = 2t$ and $e_f(0) = 3t + 1, e_f(1) = e_f(2) = 3t$.

Case (ii): $\boldsymbol{n \equiv 1 (mod\ 3)}$

Let $n = 3t + 1, t > 1$

$f(u) = 1, f(v_i') = 0$ for $i = 1$ or 2 or … n

$$f(u_i) = \begin{cases} 0, & 1 \le i \le t-1 \\ 2, & t \le i \le 2t-1 \\ 1, & 2t \le i \le 3t+1 \end{cases} \quad \text{and } f(v_i) = \begin{cases} 0, & 1 \le i \le t+2 \\ 2, & t+3 \le i \le 2t+3 \\ 1, & 2t+4 \le i \le 3t+1 \end{cases}$$

Then $v_f(0) = 2t + 2, v_f(1) = v_f(2) = 2t + 1$ and $e_f(0) = 3t + 2, e_f(1) = e_f(2) = 3t + 1$.

Case (iii): $\boldsymbol{n \equiv 2 (mod\ 3)}$

Let $n = 3t + 2, t \ge 1$

$f(u) = 1, f(v_i') = 0$ for $i = 1$ or 2 or … n

$$f(u_i) = \begin{cases} 0, & 1 \le i \le t \\ 2, & t+1 \le i \le 2t \\ 1, & 2t+1 \le i \le 3t+2 \end{cases} \quad \text{and } f(v_i) = \begin{cases} 0, & 1 \le i \le t+1 \\ 2, & t+2 \le i \le 2t+3 \\ 1, & 2t+4 \le i \le 3t+2 \end{cases}$$

Then $v_f(0) = v_f(1) = v_f(2) = 2t + 2$ and $e_f(0) = 3t + 3, e_f(1) = e_f(2) = 3t + 2$.

From all the above three cases, we see that $|v_f(i) - v_f(j)| \le 1$ and $|e_f(i) - e_f(j)| \le 1$ for all $i, j \in \{0, 1, 2\}$.

Hence f is a GMC labeling.

Illustration 3.3: GMC Labeling of graph formed by duplication of vertex v_1 of graph Helm H_{30}.

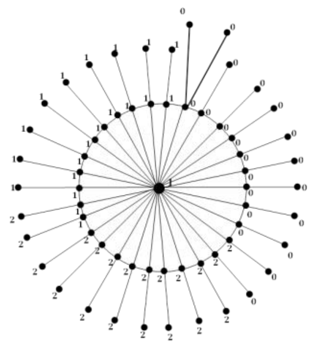

Figure 3. Duplication of vertex v_1 of H_{30} and its GMC labeling.

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Here $v_f(0) = v_f(1) = 21, v_f(2) = 20$ and $e_f(0) = 31, e_f(1) = e_f(2) = 30$.

Therefore $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$ for all $i, j \in \{0, 1, 2\}$.

Theorem 3.4: The graph formed by switching any arbitrary vertex of graph Helm H_n does not admit GMC labeling.

Proof: Let G_s represents the graph formed by switching the vertex u (apex) of graph Helm H_n. Then its vertex set $V(G_s) = \{u, u_i, v_i: 1 \leq i \leq n\}$ and its edge set $E(G_s) = \{uv_i, u_iv_i: 1 \leq i \leq n\} \cup \{u_iu_{i+1}: 1 \leq i \leq n-1\} \cup \{u_nu_1\}$. Let $|V(G_f)| = l$ and $|E(G_f)| = k$. Then $l = 2n+1, k = 3n$.

Define $f : V(G) \to \{0, 1, 2\}$ as shown below:

Case (i): $\boldsymbol{n \equiv 0 (mod\ 3)}$

Let $n = 3t, t > 1$

$$f(u) = 1$$

$$f(u_i) = \begin{cases} 0, & 1 \leq i \leq t-1 \\ 2, & t \leq i \leq 2t \\ 1, & 2t+1 \leq i \leq 3t \end{cases} \quad \text{and} \quad f(v_i) = \begin{cases} 0, & 1 \leq i \leq t+1 \\ 2, & t+2 \leq i \leq 2t \\ 1, & 2t+1 \leq i \leq 3t \end{cases}$$

Then $v_f(0) = v_f(2) = 2t, v_f(1) = 2t+1$ and $e_f(0) = 3t+2, e_f(1) = 3t-1$

and $e_f(2) = 3t-1$.

Case (ii): $\boldsymbol{n \equiv 1 (mod\ 3)}$

Let $n = 3t + 1, t > 1$

$$f(u) = 1$$

$$f(u_i) = \begin{cases} 0, & 1 \leq i \leq t-1 \\ 2, & t \leq i \leq 2t+1 \\ 1, & 2t+2 \leq i \leq 3t+1 \end{cases} \quad \text{and} \quad f(v_i) = \begin{cases} 0, & 1 \leq i \leq t+2 \\ 2, & t+3 \leq i \leq 2t+1 \\ 1, & 2t+2 \leq i \leq 3t+1 \end{cases}$$

Then $v_f(0) = v_f(1) = v_f(2) = 2t+1$ and $e_f(0) = 3t+4, e_f(1) = 3t-1$

and $e_f(2) = 3t$.

Case (iii): $\boldsymbol{n \equiv 2 (mod\ 3)}$

Let $n = 3t + 2, t \geq 1$

$$f(u) = 1$$

$$f(u_i) = \begin{cases} 0, & 1 \leq i \leq t \\ 2, & t+1 \leq i \leq 2t+1 \\ 1, & 2t+2 \leq i \leq 3t+2 \end{cases} \quad \text{and} \quad f(v_i) = \begin{cases} 0, & 1 \leq i \leq t+1 \\ 2, & t+2 \leq i \leq 2t+2 \\ 1, & 2t+3 \leq i \leq 3t+2 \end{cases}$$

Then $v_f(0) = 2t+1, v_f(1) = v_f(2) = 2t+2$ and $e_f(0) = e_f(2) = 3t+3, e_f(1) = 3t$.

From all the above three cases, we see that $|v_f(i) - v_f(j)| \leq 1$ but $|e_f(i) - e_f(j)| \nleq 1$ for all $i, j \in \{0, 1, 2\}$.

Hence G_s does not admit GMC labeling.

Illustration 3.4: Switching of vertex u (apex) of graph Helm H_{16} does not admit GMC labeling.

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https://publishoa.com
ISSN: 1309-3452

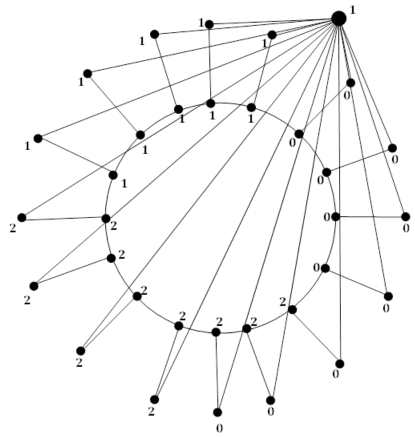

Figure 4. Switching of apex vertex u of H_{16}.

Here $v_f(0) = v_f(1) = v_f(2) = 11$ and $e_f(0) = 19, e_f(1) = 14, e_f(2) = 15$.

Therefore $\left|v_f(i) - v_f(j)\right| \leq 1$ and $\left|e_f(i) - e_f(j)\right| \nleq 1$ for all $i, j \in \{0, 1, 2\}$.

CONCLUSION

In this research article we investigated the *GMC labeling* of Helm graph H_n and also established the GMC labeling of some graph operations on Helm graph such as Fusion and Duplication. We also discussed that the operation Switching on graph Helm does not admit GMC labeling. Similar results on various graphs is an open area of research.

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