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Sum Divisor Cordial Labeling of Certain Graphs

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ABSTRACT

A *sum divisor cordial labeling* of a graph with vertex set V is bijection f from $V \to \{1, 2, \dots, |V(G)|\}$ such that an edge uv is assingned the label 1 if 2 divides $f(u) + f(v)$ and 0 otherwise; and the number of edges labeled with 0 and the number of edges labeled with 1 differ by atmost 1. A graph with sum divisor cordial labeling is called a *sum divisor cordial graph*. In this paper, we prove that splitting graph of path graph $S'(P_n)$, shadow graph of cycle union shell graph $D_2(C_n) \cup C_{(n,n-3)}$, corona product of $2K_1$ and sunlet graph $S_n \odot 2K_1$ are sum divisor cordial graphs.

Keywords: sum divisor cordial graph, sum divisor cordial labeling, splitting graph, shadow graph, sunlet graph, shell graph.

1.INTRODUCTION

All graphs considered here are simple, finite, connected and undirected. For all other standard terminology and notations we follow Harary [4]. A labeling of a graph is a map that carries the graph elements to the set of numbers, usually to the set of positive or non-negative integers. If the domain is the set of vertices then the labeling is called vertex labeling. If the domain is the set of edges then the labeling is called edge labeling. If the labels are assigned to both vertices and edges then the labeling is called total labeling. In this paper we labeled only vertices. The edge uv gets the label 1 if the label of the both end vertices are odd or even and gets the label 0 if the label of the one vertex is even and other vertex is odd. For the detailed survey of graph labeling we refer Gallian [2]. A. Lourdusamy and F. Patrick introduced the concept of sum divisor cordial labeling [5]. In this paper, we investigate the sum divisor cordial labeling behaviour of $S'(P_n), D_2(C_n) \cup C_{(n,n-3)}, S_n \odot 2K_1$.

Notation 1.1. Let $e_f(1)$ denotes the number of edges labeled with 1 and $e_f(0)$ denotes the number of edges labeled with 0.

Definition 1.2. [5] Let $G = (V(G), E(G))$ be a simple graph and $f : V(G) \to \{1, 2, 3, \dots, |V(G)|\}$ be a bijection. For each edge uv assign the label 1 if $2 / (f(u) + f(v))$ and the label 0 otherwise. The function f is called a sum divisor cordial labeling if $|e_f(0) - e_f(1)| \le 1$. A graph which admits a sum divisor cordial labeling is called a sum divisor cordial graph.

Definition 1.3. [3] The corona $G_1 \odot G_2$ of two graphs $G_1(p_1, q_1)$ and $G_2(p_2, q_2)$ is defined as the graph obtained by taking one copy of G_1 and p_1 copies of G_2 and joining the i^{th} vertex of G_1 with an edge to every vertex in the i^{th} copy of G_2.

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Definition 1.4. [9] The shadow graph $D_2(G)$ of a connected graph G is obtained by taking two copies of G, say $G^{'} and\ G^{"}$. Join each vertex $u^{'}$ in $G^{'}$ to the neighbours of corresponding vertex $u^{"}$ in $G^{"}$.

Definition 1.5. [9] For a graph G the splitting graph $S^{'}(P_n)$ of a graph G is obtained by adding a new vertex v corresponding to each vertex u of P_n such that $N(u) = N(v)$.

Definition 1.6: [1] The n-Sunlet graph is the graph on 2n vertices is obtained by attaching n-pendant edges to the cycle C_n and it is denoted by S_n.

Definition 1.7. [8] A shell graph is defined as a cycle C_n with $(n-3)$ chords sharing a common end point called the apex. Shell graph is denoted as $C_{(n,n-3)}$.

2. MAIN RESULTS

Theorem 2.1. The Graph $G = S^{'}(P_n)$ is a sum divisor cordial graph.

Proof: Let $u_1, u_2, ..., u_n$ be the vertices of path P_n and G be the graph obtained by adding new vertices $v_1, v_2, ..., v_n$ corresponding to $u_1, u_2, ..., u_n$ .Let $G = S^{'}(P_n)$ then $V(G) = \{u_i, v_i : 1 \le i \le n\}$ and $E(G) = \{u_i u_{i+1}, u_i v_{i+1}, u_{i+1} v_i : 1 \le i \le n-1\}$. Also order of the graph is 2n and size of the graph is 3n. Define $f : V(G) \to \{1, 2, 3, ..., 2n\}$ as follows:

$$f(u_1) = 1,\ f(u_2) = 4,\ f(u_3) = 6,\ f(u_4) = 7$$
$$f(u_i) = f(u_{i-4}) + 8\ ;\ i = 5, 6, 7, 8...$$
$$f(v_1) = 2,\ f(v_2) = 3,\ f(v_3) = 5,\ f(v_4) = 8$$
$$f(v_i) = f(v_{i-4}) + 8\ ;\ i = 5, 6, 7, 8...$$

Then we observed that

$$e_f(0) = e_f(1) = 3n/2 \quad \text{for} \quad n = 3, 5, 7, 9...$$

$$e_f(1) = \lceil 3n/2 \rceil \quad \text{for} \quad n = 2, 4, 6, 8...$$

$$e_f(0) = \lfloor 3n/2 \rfloor$$

So, we have the difference between the number of edges labeled as 0 and the number of edges labeled as 1 is less than or equal to 1 (i.e). $|e_f(0) - e_f(1)| \le 1$. Hence the graph $G = S^{'}(P_n)$ is a sum divisor cordial graph.

EXAMPLE 1. A sum divisor cordial labeling of $S^{'}(P_6)$ is shown in fig.2.1.2

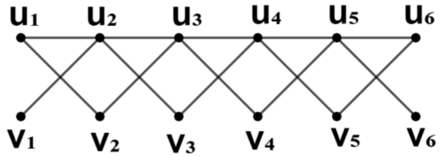

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Figure 2.1.1

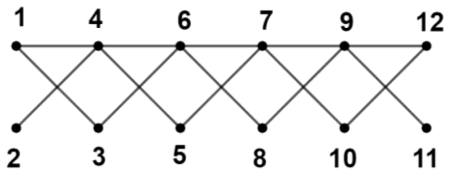

Figure 2.1.2

Theorem 2. 2. The graph $G = D_2\left(C_n\right) \cup C_{(n,n-3)}$ is a sum divisor cordial graph.

Proof: Let $u_1, u_2, ..., u_n$ be the vertices of the first copy of the cycle C_n, $v_1, v_2, ..., v_n$ be the vertices of the second copy of the Cn. and $u_1, u_2, ..., u_n$ be the vertices of the shell graph. Let $G = D_2\left(C_n\right) \cup C_{(n,n-3)}$. Then $V\left(G\right) = \left\{u_i, v_i : 1 \le i \le n\right\}$ and $E\left(G\right) = \left\{v_1u_n, v_1v_n, u_1v_n, u_1u_n\right\} \cup \left\{v_iv_{i+1}, u_iu_{i+1}, u_iv_{i+1} ; 1 \le i \le n-1\right\} \cup \left\{u_iv_{i-1} ; 2 \le i \le n\right\} \cup \left\{u_1u_i ; 3 \le i \le n-1\right\}$ Also, G is of the order $2n$ $\left(n \ge 5\right)$ and size $4n + \left(n-3\right)$. Let m be the number of vertices $\left(u_i\right)$ $\left(3 \le i \le n\right)$ adjacent to u_1. Define $f : V\left(G\right) \to \left\{1, 2, 3, ..., 2n\right\}$ as follows:

$$f\left(u_1\right) = 1,\ f\left(u_2\right) = 4,\ f\left(u_3\right) = 6,\ f\left(u_4\right) = 7$$
$$f\left(u_i\right) = f\left(u_{i-4}\right) + 8 ; i = 5, 6, 7, 8...$$

$$f\left(v_1\right) = 2, f\left(v_2\right) = 3,\ f\left(v_3\right) = 5,\ f\left(v_4\right) = 8$$
$$f\left(v_i\right) = f\left(v_{i-4}\right) + 8 ; i = 5, 6, 7, 8...$$

We observe that

Case (i)

$$n \equiv 1\left(\text{mod } 2\right) \quad n \ge 5$$
$$e_f\left(0\right) = e_f\left(1\right) = 2n + m/2 ; m = even$$

Case (ii)

$$n \equiv 2\left(\text{mod}\, 4\right) \quad n \ge 6$$
$$e_f\left(1\right) = 2n + \lceil m/2 \rceil ; m = odd$$
$$e_f\left(0\right) = 2n + \lfloor m/2 \rfloor$$

Case (iii)

$$n \equiv 0\left(\text{mod } 4\right) \quad n \ge 8$$
$$e_f\left(1\right) = 2n + \lfloor m/2 \rfloor ; m = odd$$
$$e_f\left(0\right) = 2n + \lceil m/2 \rceil$$

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Thus we have $\left|e_f(0)-e_f(1)\right| \le 1$. Hence the graph G is a sum divisor cordial graph.

EXAMPLE 2. A sum divisor cordial labeling of $D_2(C_6) \cup C_{(6,3)}$ is shown in fig.2.2.2

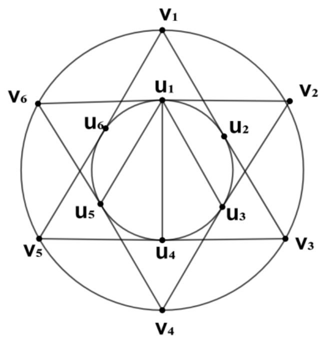

Figure 2.2.1

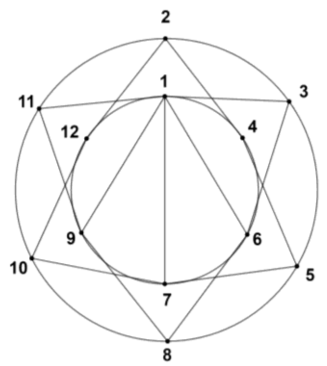

Figure 2.2.2

Theorem 2.3. The graph $G = S_n \odot 2K_1$ is a sum divisor cordial graph.

Proof: Let $u_1, u_2, \ldots, u_n$ be the vertices of the C_n *and* $v_1, v_2, \ldots, v_n$ be the pendent vertices of the each vertex of the cycle C_n. Let $w_1, w_2, \ldots, w_n$ be the vertices joining to the each pendent vertex of the Sunlet graph. Let $G = S_n \odot 2K_1$. $V(G) = \{u_i, v_i : 1 \le i \le n\} \cup \{w_i : 1 \le i \le 2n\}$ and. $E(G) = \{u_i u_{i+1} : 1 \le i \le n-1\} \cup \{u_1 u_n\} \cup \{u_i v_i : 1 \le i \le n\} \cup \{v_i w_{2i-1}, v_i w_{2i} : 1 \le i \le n\}$. Also the order of the graph is 4n and size of the graph is 4n. Define $f : V(G) \to \{1, 2, 3, \ldots, |V(G)|\}$ as follows:

$$f(u_1) = 1,\ f(u_i) = f(u_{i-1}) + 4\ ;\ 2 \le i \le n$$
$$f(v_1) = 2,\ f(v_i) = f(v_{i-1}) + 4\ ;\ 2 \le i \le n$$
$$f(w_1) = 3,\ f(w_2) = 4$$
$$f(w_i) = f(w_{i-1}) + 4 \qquad \text{if } i \text{ is odd}\ ;\ 3 \le i \le 2n-1$$
$$f(w_i) = f(w_{i-1}) + 4 \qquad \text{if } i \text{ is even}\ ;\ 4 \le i \le 2n$$

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Here we observe that $e_f(0) = e_f(1) = 2n$. So, we have $|e_f(0) - e_f(1)| \le 1$. Hence the graph G is a sum divisor cordial graph.

EXAMPLE 3. A sum divisor cordial labeling of $S_6 \odot 2K_1$ is shown in fig.2.3.2

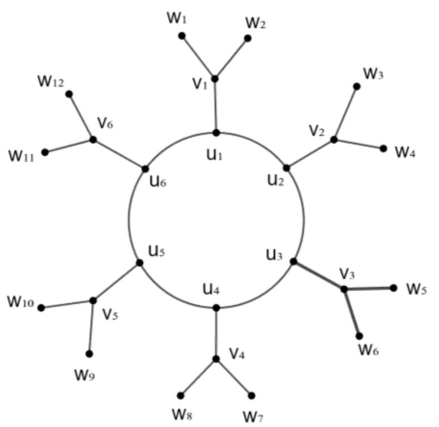

Figure 2.3.1

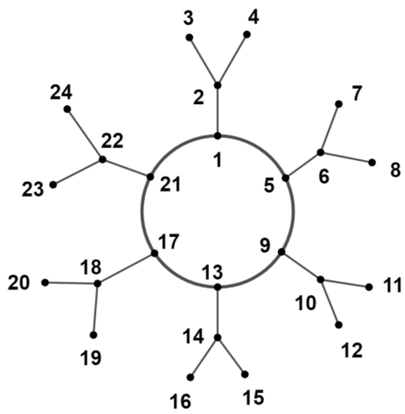

Figure 2.3.2

3. CONCLUSION

Here, we proved that $S^{'}(P_n), D_2(C_n) \cup C_{(n,n-3)}, S_n \odot 2K1$ are sum divisor cordial graphs. Shadow graph of comb, shuriken graph, globe graph are under investigation for sum divisor cordial labelling.

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