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Upper Domatic Number of Certain Graphs

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Abstract

Let $G(V,\ E)$ be a graph with vertex set $V(G)$ and edge set $E(G)$. The maximum order of the partitions of the vertex set represented by $\{V_1, V_2, \dots, V_k\}$, is called *upper domatic number* where each class of partition dominates one another (i.e.) V_i dominates V_j or V_j dominates V_i or both $(i \neq j)$. U*pper domatic number* is denoted by $D(G)$. In this paper we determine the upper domatic number of certain graphs such as *double fan graph* DF_n, *friendship graph* F_n and *helm graph* H_n.

AMS Subject Classification: 05C62, 05C69

Key Words and Phrases: upper domatic number, double fan graph, friendship graph, helm graph.

1. Introduction

Cockayne and Hedetniemi [1] in 1977 defined the domatic number to be the maximum order of a partition of the vertices of a graph into dominating sets. The subset D of the vertex set V is called the dominating set, if every vertices in V/D is adjacent to atleast one vertex in D. The number of vertices of a minimum dominating set of a graph G is called the domination number of G, denoted by $\Upsilon(G)$. Investigating the relation between the sets in a vertex partition, Teresa W. Haynes [5] introduced the *upper domatic number* $D(G)$, equals the maximum order k of a vertex partition $\Pi = \{V_1, V_2, \dots, V_k\}$ such that for all i, j, $1 \leq i < j \leq k$, either $V_i \rightarrow V_j$ or $V_j \rightarrow V_i$ or both. A vertex partition Π meeting this condition is called an *upper domatic partition* and an *upper domatic partition* of order $D(G)$ is called a $D - partition$ of G. The graph considered in this paper are simple, connected and undirected graph. Let $G(V,\ E)$ be a graph for two disjoint set of vertices S_1 and S_2, we say that S_1dominates S_2 denoted by $S_1 \rightarrow S_2$. If every vertex in $S_1$is adjacent to or dominated by every other vertex in S_2. In this paper, we determine the *upper domatic number* of certain graphs such as *double fan graph* DF_n, *friendship graph* F_n and *helm graph* H_n.

2. Premilinaries

Definition 2.1[1]: *Dominating set* of graph $G(V,\ E)$ is defined as the subset of the vertex set $V\ (G)$ denoted by S such that every vertices of $V(G) - S$ is adjacent to atleast one vertex of S.

Definition 2.2[5]: *Upper domatic number* $D(G)$ which equals the maximum order k of a vertex partition $\{\mathrm{V}, V_{2,} \dots, V_k\}$ such that for every $i,\ j$, $1 \leq i < j \leq k$, either V_i dominates V_j or V_j dominates V_i or both $(i \neq j)$.

Definition 2.3[4]: The *double fan graph* DF_n consists of two fan graphs that have a common path. (i.e.) $DF_n = P_n + K_2$

Definition 2.4[2]: The *Helm Graph* H_n is the graph obtained from a wheel graph by adjoining a pendant edge to each node of the cycle.

Definition 2.5[3]: The *Friendship Graph* F_n is a planar undirected graph constructed by joining n copies of the 3-cycle graph $C_3$with a common vertex.

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3. Main Results

***Theorem 3.1*:**

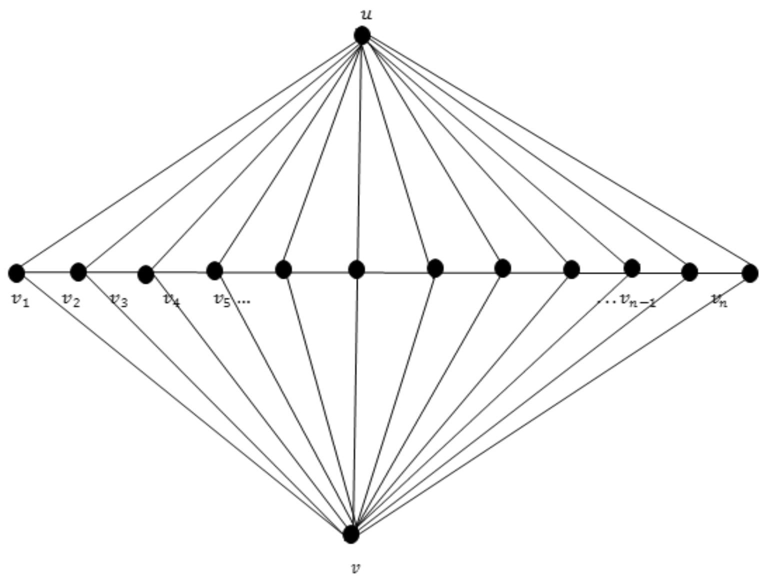
Upper domatic number of double fan graph DF_n for $n \geq 2$ is $D(G) = 2$.

Proof:

Let $G(V,\ E)$ be a double fan graph and V be the vertex set defined as $V(G) = \{u,\ v,\ v_1,\ v_2,\ v_3, \dots,\ v_n\}$ such that $|V| = n + 2$ and E be the Edge set defined as $E(G) = \{u\, v_i / \, 1 \leq i\ \leq n\} \cup \{v\, v_i \,/\, 1 \leq i\ \leq n\} \cup \{v_i\, v_{i+1} \,/\, 1 \leq i\ \leq n - 1\}$ such that $|E| = 3n - 1$.

Let us consider the partition of the vertex set V(G) as $S_1 = \{u, v\}$ and $S_2 = \{v_i / \, 1 \leq i\ \leq n\}$ then $V - S_1 = \{v_i / \, 1 \leq i\ \leq n\}$ and $V - S_2 = \{u, v\}$. Since S_1 is adjacent to all the vertices of $V - S_1$, we say that S_1dominates S_2, similarly S_2 is adjacent to all the vertices of $V - S_2$, we say that S_2 dominates S_1. Therefore, we conclude that S_1 dominates S_2 and S_2 dominates S_1 (i.e.) $S_1 \rightarrow\ S_2 \rightarrow S_1$. Since there are only two partitions S_1 and S_2 of the vertex set $V(G)$, the maximum order of the partitions is 2. Hence, upper domatic number of the double fan graph is 2.

Therefore, $D(G) =\ 2$.



***Theorem 3.2*:**

Upper domatic number of helm graph DF_n for $n \geq 2$ is $D(G) =\ 2$.

Proof:

Let $G(V,\ E)$ be a helm graph of order n with the vertex set $V(G) = \{u_0,\ u_1, u_{2,} \dots,\ u_{n,}\ v,\ v_1,\ v_{2,}\ v_{3,} \dots,\ v_n\}$ such that $|V| = 2n + 1$ and the edge set $E(G) = \{u_0\, u_i \,/\, 1 \leq i\ \leq n\} \cup \{\, u_i\, v_i \,/\, 1 \leq i\ \leq n\} \cup \{u_i\, u_{i+1} \,/\, 1 \leq i\ \leq n - 1\} \cup \{u_n\, u_1\}$, such that$|E| = 3n$.

Let us consider the partitions set $S_1 = \{u_1,\ u_{2,}\ \dots,\ u_{n,}\}$ and $S_2 = \{u_0,\ v_1,\ v_{2,}\ \dots,\ v_n\}$. All the vertices in the set S_1 is adjacent to atleast one vertex in the set S_2, also vice versa. Therefore, we conclude that the set S_1 dominates the set S_2 and the set S_2 dominates the set S_1 (i.e.) $S_1 \rightarrow S_2 \rightarrow S_1$. Since there are only two partitions S_1 and S_2 of the vertex set $V(G)$, the maximum order of the partitions is 2. Hence, the upper domatic number of the helm graph is 2.

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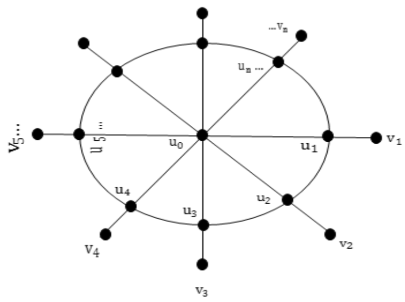
Therefore, $D(G) = 2$.



***Theorem 3.3*:**

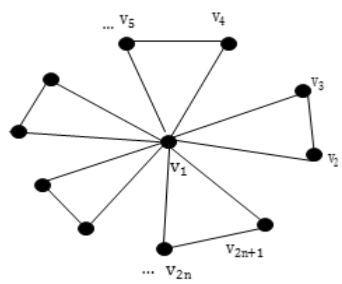
Upper domatic number of friendship graph F_n for $n \geq 2$ is $D(G) = 2$.

Proof:

Let $G(V, E)$ be a friendship graph of order n with the vertex set $V(G) = \{v_1, v_2, v_3, \ldots, v_{2n+1}\}$ with v_1 as the centre vertex and the edge set $E(G) = \{v_1 v_i / 2 \leq i \leq 2n+1\} \cup \{v_{2i} v_{2i+1} / 1 \leq i \leq n\}$, such that $|E| = 3n$.

Let us consider the partitions set $S_1 = \{v_{1,}\}$ and $S_2 = \{v_2, v_{3,} \ldots, v_{2n+1}\}$, Since v_1 is the centre vertex it is adjacent to all other vertices of the vertex set $V(G)$ (i.e.) the vertex in the set S_1 is adjacent to all the vertices in the set S_2. Hence, we conclude that the set S_1 dominates the set S_2 and the set S_2 dominates the set S_1 (i.e.) $S_1 \to S_2 \to S_1$, since there are only two partitions S_1 and S_2 of the vertex set $V(G)$, the maximum order of the partitions is 2. Hence, upper domatic number of the friendship graph is 2.

Therefore, $D(G) = 2$



4. Conclusion:

In this paper we have deduced the upper domatic number of *double fan graph* DF_n, *friendship graph* F_n, *helm graph* H_n. To investigate the upper domatic number of similar graphs is an open area in research.

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