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Detour D – Eccentric Domination in Graphs

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ABSTRACT

A dominating set S $\subset$V is said to be a detour eccentric dominating set in *G (V, E)* if for every u in V – S, there exists at least a detour eccentric vertex v of u in S. A dominating set D is said to be a *D* – eccentric dominating set in a graph G if for every u in V – D, there exists at least a *D* – eccentric vertex v of u in D. In this paper, the detour *D* – eccentric dominating set and its numbers are introduced. The bounds on detour *D* – eccentric numbers are obtained for some well-known graphs. Theorems related to the above concepts are stated and proved.

2010 Mathematics Subject Classification: 05C12, 05C69

Key words: Detour *D* – eccentric vertex, Detour *D* – eccentric vertex set, Detour *D* – eccentric dominating set, Detour *D* – eccentric domination number.

1. INTRODUCTION

This section delineates the various new ideas of the different writers who bring the new definition to their ideas that have been from 1962 to present day. It is that O. Ore has designated his idea as dominating set and domination number in 1962 [9], followed by him in 1998, T. W. Haynes, et.al., deliberated various dominating parameters [5]. In 2010, T.N. Janakiraman, et.al., illustrated eccentric domination in graphs[6]. Detour eccentric domination in graphs were developed by A. Mohamed Ismayil and R. Priyadharshini in 2019 [8]. *D*- Eccentric domination in graphs were determined by A. Prasanna and N. Mohamedazarudeen in 2021 [10]. Article [8, 10] persuade us to think about the detour *D*-eccentric domination in graphs. In this paper, detour *D*-eccentric dominating set as well as detour *D*-eccentric domination numbers are introduced.

2. PRELIMINARIES

In this section, Preliminary definitions are given to under this paper very well. Many of the researcher in mathematics knows, what is graph, subgraph, order, size, neighbourhood, degree, path, length and so on. Here, the definition of *D* – length and detour *D* – length are given. Also, detour *D* – eccentricity of a vertex and different known graphs are defined.

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Definition 2.1: The detour *D*-length of a $v_0 - w_0$ path p is elucidated as $l_D^D(p) = D(v_0, w_0) + deg(v_0) + deg(w_0) + \sum deg(s_0)$ where $\sum \deg(s_0)$ *is the* sum runs over all intermediate vertices s_0 of p. The detour *D*- distance $d_D^D(v_0, w_0) = max \{ l_D^D(p) \}$ = the maximum is taken over all $v_0 - w_0$ paths in *G*.

Example 2.1:

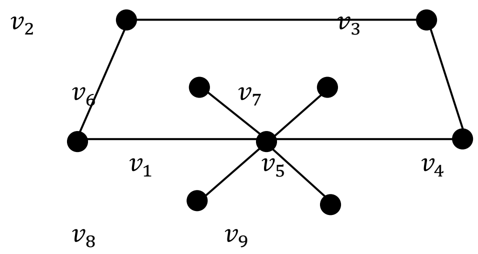

Figure 2.1

From the graph given in figure 2.1, there are two paths between v_1 and v_4, p_1: v_1 - v_5 - v_4, another one is p_2: v_1 - v_2 - v_3 - v_4. Length of p_1 is 2 and length of p_2 is 3. Here $d(v_1, v_4) = 2$, $D(v_1, v_4) = 3$ and $d^D(v_1, v_4) = 11$ and $d_D^D(v_1, v_4) = 12$.

Definition 2.2: The detour *D*-eccentricity of avertex w_0 is defined by $e_D^D(w_0) = max \{d_D^D(v_0, w_0) / v_0 \in V\}$

Definition 2.3: The detour *D*-radius in a graph *G (V, E)* is defined and denoted by $r_D^D(G) = min \{ e_D^D(w_0): w_0 \in V \}$. The detour *D*-diameter is defined and denoted by $d_D^D(G) = max \{ e_D^D(w_0): w_0 \in V \}$.

Definition 2.4: The vertex w_0 in *G* is a detour *D*-central vertex if $r_D^D(G) = e_D^D(w_0)$ and the detour *D*-center $C_D^D(G)$ is the set of all central vertices.

Definition 2.5: The detour *D*-peripheral of *G*, $P_D^D(G) = e_D^D(G)$. *V* is a detour *D*-peripheral vertex if $e_D^D(w_0) = d_D^D(G)$. The detour *D*-periphery $P_D^D(G)$ is the set of all peripheral vertices.

Definition 2.6: For a vertex w_0, each vertex at a detour *D*-distance $e_D^D(w_0)$ from w_0 is a detour *D*-eccentric vertex of w_0. detour *D*-eccentric set of a vertex w_0 is defined as $E_D^D(w_0) = \{ v_0 \in V / d_D^D(w_0) = e_D^D(w_0) \}$ or any vertex v_0 for which $d_D^D(v_0, w_0) = e_D^D(w_0)$ is called detour *D*-eccentric vertex of w_0.

Definition 2.7: The Wagner graph is a 3- regular graph with | V | = 8 and | E | = 12. It is a 8 – vertex M*ö*bius ladder graph.

Definition 2.8: The Frucht graph is a 3- regular graph with | V | = 12 and | E | = 18 and no non trivial symmetries.

Definition 2.9: The Franklin graph is a 3- regular graph with | V | = 12 and | E | = 18.

In this paper, as it were non trivial basic associated undirected graphs are considered and for all the other vague terms one can refer [2, 3].

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3. DETOUR *D*-ECCENTRIC VERTEX SET AND DETOUR *D*-ECCENTRIC DOMINATING SET

In this section, the detour *D* - eccentric vertex set, detour *D* - eccentric dominating sets and its numbers are defined with suitable examples.

Definition 3.1: Let $W \subset V(G)$ be a set of vertices in a graph $G(V, E)$, then *W* is said to be a detour *D*-eccentric vertex set of *G* if for every vertex $w_0 \in V - W$ has at least one vertex $v_0 \in W$ such that $v_0 \in E_D^D(w_0)$. A detour *D*- eccentric vertex set *W* of *G* is called minimal detour *D*-eccentric vertex set, if no proper subset W' of *W* is a detour *D*-eccentric vertex set of *G*. The minimum cardinality of a minimal detour *D*-eccentric vertex set of *W* is called the detour *D*-eccentric number and is denoted by e_D^D *(G)* and simply denoted by e_D^D. The maximum cardinality of a minimal detour *D*-eccentric vertex set is called the upper detour *D*-eccentric number and is denoted by E_D^D *(G)* and simply denoted by E_D^D.

Example 3.1: The detour *D*-eccentric vertex set and its numbers are defined in a graph $G(V, E)$ with suitable example as given below

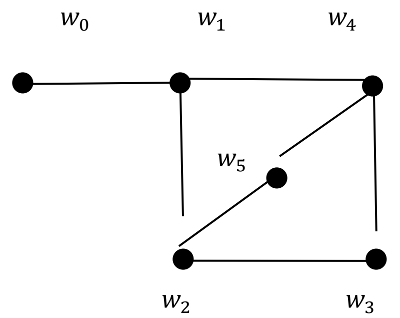


Figure 3.1

In a graph $G(V, E)$ as given figure 3.1, the detour *D*-eccentricity of w_0, w_1, w_2, w_3, w_4 and w_5 are $e_D^D(w_0) = e_D^D(w_3) = e_D^D(w_5) = 19$, $e_D^D(w_2) = e_D^D(w_4) = 16$ and $e_D^D(w_1) = 17$ respectively. The detour *D*-eccentric sets of w_0, w_1, w_2, w_3, w_4 and w_5 are $E_D^D(w_0) = E_D^D(w_1) = \{w_3, w_5\}$ and $E_D^D(w_2) = E_D^D(w_3) = E_D^D(w_4) = E_D^D(w_5) = \{w_0\}$ respectively. Then the sets $W_1 = \{w_0, w_3\}$, $W_2 = \{w_2, w_3, w_4, w_5\}$ etc., are some detour *D*-eccentric vertex sets of $G(V, E)$. The detour *D*-eccentric number $e_D^D = 2$ and upper *D*-eccentric number $E_D^D = 4$.

Note 3.1: v_0 is a detour *D*-eccentric vertex of w_0, then $v_0 \in E_D^D(w_0)$.

Observations 3.1:

1) Every superset of a detour *D*-eccentric set is detour a *D*-eccentric vertex set.
2) The subset of a detour *D*-eccentric vertex set need not be a detour *D*-eccentric vertex set.
3) In a graph $G(V, E)$, $e_D^D(G) \leq E_D^D(G)$

Definition 3.2: A dominating set $D \subseteq V(G)$ of a graph $G(V, E)$ is said to be a detour *D*-eccentric dominating set if for every vertex $w_0 \in V - D$, has at least one vertex $v_0 \in D$ such that $v_0 \in E_D^D(w_0)$. A detour *D*- eccentric dominating set *D* is a minimal detour *D*-eccentric dominating set if there exists a subset $D' \subset D$ which is not a detour *D*-eccentric dominating set. The minimum cardinality of a minimal detour *D*-eccentric domination set of *D* is called the detour *D*-eccentric domination number and is denoted by γ_{Ded}^{D}. The maximum cardinality of a minimal detour *D*-eccentric dominating set of *D* is called the upper detour *D*-eccentric dominating set and is denoted by $\Gamma_{\mathrm{Ded}}^{\mathrm{D}}$ (G).

Example 3.2: Consider the Fig 3.1, $D_1 = \{ w_0, w_3, w_5\}$, $D_2 = \{ w_1, w_2, w_3, w_4, w_5\}$ etc., are some detour *D*-eccentric dominating sets. The detour *D*-eccentric domination number is γ_{Ded}^{D} = 3 and upper detour *D*-eccentric domination number is $\Gamma_{\mathrm{Ded}}^{\mathrm{D}}$ = 5.

Remark 3.1: If *W* is a minimum detour *D*-eccentric vertex set and D is a dominating set of *G* then $D \cup W$ is a detour *D*-eccentric dominating set of *G*.

Observations 3.2:

(i) For any connected graph G, $\gamma(G) \leq \gamma_{Ded}^{D}(G) \leq \Gamma_{\mathrm{Ded}}^{\mathrm{D}}$.

(ii) Every detour *D*-eccentric dominating set is a dominating set but the converse is not true.

(iii) If $r_D^D(G) = d_D^D(G)$, then $\gamma(G) = \gamma_{Ded}^{D}(G)$.

(iv) For any connected graph, $\gamma_{Ded}^{D}(G) \leq \gamma(G) + e_D^D(G)$.

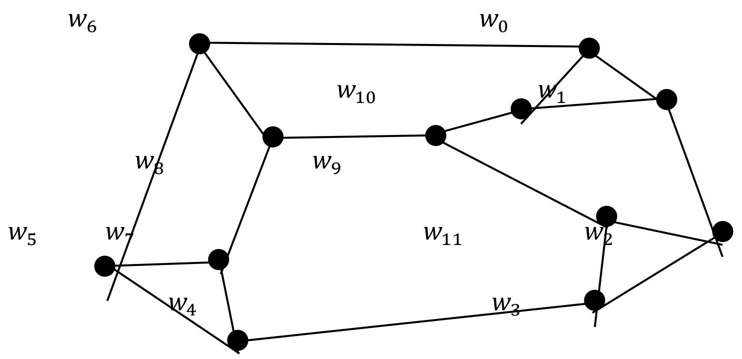
(v) If *G* is disconnected then $\gamma(G) = \gamma_{Ded}^{D}(G)$, since vertices from different components are detour *D*-eccentric to each other and if *G* is disconnected graph and v_0, w_0 are in different components then $d^{D}(v_0, w_0) = \infty$.

4. SOME RESULTS ON DETOUR *D*-ECCENTRIC NUMBERS OF FRUCHT GRAPH , FRANKLIN GRAPH AND WAGNER GRAPH

In this section, the detour *D*-eccentric numbers of Frucht graph , Franklin graph and Wagner graph are obtained.

Result 4.1: Detour *D* - eccentric number of Frucht graph G is unity, that is $e_D^D(G) = 1$. Alternatively, every singleton set in a Frucht graph G is detour *D*- eccentric vertex set of G.

Solution:



Frucht graph 4.1

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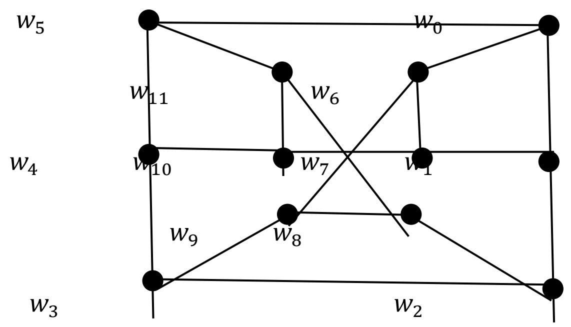
Let G (V,E) be a Frucht graph and $d_D^D(u,v)$ be the detour *D*- distance from u to v. Let u, v be any two vertices in G. Obviously, $d_D^D(u,v) = e_D^D(u) = r_D^D(u) = diam_D^D(u)$. Hence, $E_D^D(u_i)$ = V – $\{u_i\}$, i = 1 to 12. For any vertex $\{u_i\} \subset$ V is a detour *D* - eccentric vertex set of G. Therefore e_D^D *(G)* = 1.

Result 4.2: Detour *D* - eccentric number of Franklin graph G is two, that is e_D^D *(G)* = 2.

Solution:

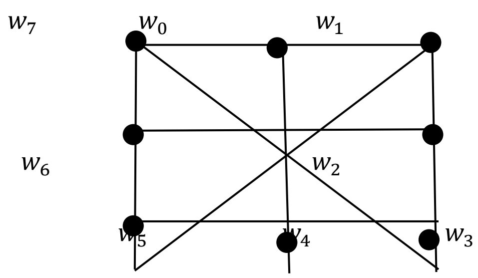


Franklin graph 4.2

Let G (V,E) be a Franklin graph and $d_D^D(u,v)$ be the detour *D*- distance from u to v. Let u, v be any two vertices in G. Therefore $d_D^D(u,v)$ = 43 (or) 47. But $e_D^D(u) = r_D^D(u) = diam_D^D(u)$ = 47. Here $E_D^D(u_i)$, i = 1 to 12 have two different sets either $S_1 = \{w_0, w_2, w_4, w_7, w_9, w_{11}\}$ or $S_2 = \{w_1, w_3, w_5, w_6, w_8, w_{10}\}$. Hence $e_D^D(G)$ = {u, v} such that u ∈ S_1 and v ∈ S_2 is a detour *D*-eccentric vertex set of G. Therefore e_D^D *(G)* = 2.

Result 4.3: Detour *D* - eccentric number of Wagner graph G is three, that is e_D^D *(G)* = 3. Alternatively, the neighbourhood of every vertex in Wagner graph is a detour *D*-Eccentric dominating set.

Solution:



Wagner graph 4.3

Let G (V,E) be a Wagner graph and $d_D^D(u,v)$ be the detour *D*- distance from u to v. Let u,v be any two vertices in G. Therefore $d_D^D(u,v)$ = 31 (or) 27 in Wagner graph. But $e_D^D(u) = r_D^D(u) = diam_D^D(u)$ = 31. Let E (u_i) be the eccentric set of a vertex u_i .

Here $E_D^D(u_i) = V - [\, E\,(u_i) \cup \{u_i\}] = N\,(u_i)$, for i = 1 to 8 is a detour *D* - eccentric vertex set of u_i and also a detour *D* - eccentric vertex set of G. Since G is a 3 – regular graph. Therefore e_D^D *(G)* = 3.

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5. SOME RESULTS ON DETOUR *D*-ECCENTRIC DOMINATION NUMBERS OF FRUCHT GRAPH , FRANKLIN GRAPH AND WAGNER GRAPH

In this section, we explained about the detour *D*-eccentric domination numbers of Frucht graph, Franklin graph and Wagner graph.

Result 5.1: Detour *D* – eccentric domination number of Frucht graph G is three, that is γ_{Ded}^{D} = 3.

Solution:

From Frucht graph 4.1, one of the minimum dominating set is D = $\{w_0, w_7, w_{11}\}$. By result 4.1, every singleton set of a Frucht graph G is detour *D* - eccentric vertex set of G. Hence the set D is a detour *D*- eccentric dominating set of G. Therefore γ_{Ded}^{D} = 3.

Remark 5.1:

In Frucht graph r(G) = γ (G) = 3 and diam(G) = $\Gamma(G)$ = 4.

Result 5.2: Detour *D* – eccentric domination number of Franklin graph G is Four, that is γ_{Ded}^{D} = 4.

Solution: Let G (V, E) be a Franklin graph and $d_D^D(u, v)$ be the detour *D*- distance from u to v. Let u,v be any two vertices in G. Therefore $d_D^D(u, v)$ = 43 (or) 47 in Franklin graph. But $e_D^D(u)$ = $r_D^D(u)$ = $diam_D^D(u)$ = 47. Here $E_D^D(u_i)$, i = 1 to 12 have two different sets either $S_1 = \{ w_0, w_2, w_4, w_7, w_9, w_{11}\}$ or $S_2 = \{ w_1, w_3, w_5, w_6 , w_8, w_{10}\}$. Hence $e_D^D(G)$ = {u,v} such that u ∈ S_1 and v ∈ S_2 is a detour *D* - eccentric vertex set of G. Therefore e_D^D *(G)* = 2. From Franklin graph 4.2, one of the minimum dominating set is D = $\{w_0, w_1, w_2, w_{10}\}$. Here D = (D∩ S_1)∪(D∩ S_2) is a detour *D*- eccentric dominating set of G. Therefore γ_{Ded}^{D} = 4.

Remark 5.2:

In Franklin graph with twelve vertices, d (u,v) = 1 (or) 3 then d_D^D (u,v) = 47 and d (u, v) = 2 then d_D^D (u,v) = 43.

Result 5.3: Detour *D* – eccentric domination number of Wagner graph G is three, that is γ_{Ded}^{D} = 3.

Solution: By result 4.3, N (u_i) is detour *D* - eccentric vertex set and also a dominating set of G. Hence N (u_i) is detour *D*- eccentric dominating set. Therefore γ_{Ded}^{D} = 3.

Remark 5.3:

In the Wagner graph G, u, v ∈ G,

(i) If d (u,v) = 1, then d_D^D (u,v) = 31 and if d (u,v) = 2, then d_D^D (u,v) = 27.

(ii) If e (v) =2 then e_D^D (v) = 31 in Wagner graph.

6. BOUNDS ON DETOUR *D*-ECCENTRIC DOMINATION IN STANDARD GRAPHS

In this section, we discussed about bounds on detour *D*-eccentric domination numbers in some standard graphs.

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Observations 6.1:

(i) $\gamma_{Ded}^{D}(K_n) = 1, n \geq 2$

(ii) $\gamma_{Ded}^{D}(K_{1,n}) = 2, n \geq 3$

(iii) $\gamma_{Ded}^{D}(K_{m,n}) = 2, n \geq 1, m \geq 1$

(iv) $\gamma_{Ded}^{D}(W_n) = 1. n \geq 4$

Remark 6.1:

In star graph with n vertices, if d (u,v) = 2 then d_D^D (u,v) = n+3 and if d (u,v) = 1 then d_D^D (u,v) = n+1

Proposition 6.1: In a path with four vertices, the domination number is equal to the detour *D*-eccentric domination number.

Theorem 6.1: In a path (P_r) of order $r>2, n=1,2,....\frac{r-2}{3}$

$$\gamma_{Ded}^{D}(P_r) = \begin{cases} \left\lceil \frac{r}{3} \right\rceil + 1, & \text{if } r = 3n, \\ \left\lceil \frac{r}{3} \right\rceil, & \text{if } r = 3n+1, \\ \left\lceil \frac{r}{3} \right\rceil + 1, & \text{if } r = 3n+2. \end{cases}$$

Proof: ***Case(i): r = 3n.***

Let $w_1, w_2, w_3, \ldots, w_{3m}$ represent the path P_r and has all the peripheral vertices. $D = \{ w_2, w_5, w_8, \ldots, w_{3n-1}\}$ is the only γ - set of P_r, but not $\gamma_{Ded}^{D}(P_r)$. That is $\gamma_{Ded}^{D}(P_r)$ is $D' = \{ w_1, w_4, w_7, \ldots, w_{3n}\}$ where $|D'| = n+1 = \gamma(P_r) + 1$. Therefore, $\gamma_{Ded}^{D}(P_r) = \left\lceil \frac{r}{3} \right\rceil + 1$.

Case(ii): r = 3n + 1

$D = \{ w_1, w_4, w_7, \ldots, w_{3n-2}, w_{3n+1}\}$ is the least dominating set P_r has two peripheral vertices. Hence, $\gamma(P_r) = \gamma_{Ded}^{D}(P_r) = \left\lceil \frac{r}{3} \right\rceil$.

Case(iii): r = 3n + 2

$D = \{ w_2, w_5, w_8, \ldots, w_{3n+2} \}$ has end vertices w_{3n+2} and it is not a detour *D*-eccentric dominating set . Hence, $D \cup \{w_1\}$ is a minimum detour *D*-eccentric dominating set. Therefore $\gamma_{Ded}^{D}(P_r) = \gamma(P_r) + 1 = \left\lceil \frac{r}{3} \right\rceil + 1$.

Remark 6.2: In a path (P_r) of order $r=2, \gamma_{Ded}^{D}(P_r) = 1$.

Theorem 6.2: In a cycle graph (C_n) of order n > 2, n = 1,2,…

$\gamma_{Ded}^{D}(C_n) = \left\lceil \frac{n}{3} \right\rceil$ for n ≥ 3.

Proof: In a cycle graph C_n, let u ∈ C_n, the detour D – distance of u and adjacent vertices of u is always greater than the detour D – distance of u and non - adjacent vertices of u. Therefore d_D^D (u,v) > d_D^D (u,w) since v belongs to N (u) and w does not belongs to N (u). For any two vertices u,v in cycle graph, then max{ $d_D^D(u,v)$/u,v ∈ C_n} = $e_D^D(u)$ = $r_D^D(u)$ = $diam_D^D(u)$ = 3n - 1. let γ (C_n) set be the minimum dominating sets of cycle graph. This γ (C_n) set is detour *D*-eccentric vertex set and a detour *D*-eccentric dominating set of C_n. Hence $\gamma_{ed}^D(C_n) = \left\lceil \frac{n}{3} \right\rceil$, n ≥ 3.

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