JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 3182-3187
https://publishoa.com
ISSN: 1309-3452

On Intutionistic I – Open Sets In Intutionistic Topological Spaces

J. Arul Jesti[1] , P. Suganya [2]

[1]Assistant Professor, [2]Research Scholar, Reg.No.19222212092015,
Department of Mathematics, St. Mary's College(Autonomous),
(Affiliated to Manonmaniam Sundaranar University, Abishekapatti -627012, Tirunelveli)
Thoothukudi-1,TamilNadu, India
[1]aruljesti@gmail.com
[2]suganyaprince20@gmail.com

Received 2022 March 25; **Revised** 2022 April 28; **Accepted** 2022 May 15.

Abstract

The purpose of this paper is to provide the notion of Intutionistic i-open sets in Intutionistic topological spaces and study the relation with some existing Intutionistic open sets. Additionally, we expounded some properties of Intutionistic i-open sets in Intutionistic topological spaces.

MSC 2010 : 54A05

Keywords Intutionistic i-open sets, Intutionistic i-closed sets, Intutionistic i-interior, Intutionistic i-closure, Intutionistic i-neighborhood

1. Introduction

The idea of intutionistic fuzzy sets was introduced by Atanassov[1]. Notion of membership and non membership were discovered by Coker [3] in intuitionistic fuzzy topological spaces, subsequently he modified the crisp sets in entire forms. Later, Coker [5] introduced the intuitionistic topological spaces using intuitionistic sets. This paper is an attempt to define the conception of intutionistic i-open sets in intutionistic topological spaces and some characterizations of intutionistic i-open sets are discussed. Besides, we relate intutionistic i-open sets with other existing intutionistic open sets in intutionistic topological spaces.

2. Preliminaries

Definition 2.1 [2]. Let $\mathcal{K}$ be a non-empty set. An intuitionistic set(IS for short) $\mathcal{H}$ is an object having the form $\mathcal{H}$ = $<\mathcal{K}, \mathcal{H}_1, \mathcal{H}_2>$ where $\mathcal{H}_1, \mathcal{H}_2$ are subsets of $\mathcal{K}$ satisfying $\mathcal{H}_1 \cap \mathcal{H}_2 = \emptyset$. The set $\mathcal{H}_1$ is called the set of members of $\mathcal{H}$, while $\mathcal{H}_2$ is called set of non members of $\mathcal{H}$.

Definition 2.2 [2]: Let $\mathcal{K}$ be a non-empty set and $\mathcal{H}$ and $\mathcal{G}$ are intuitionistic set in the form $\mathcal{H} =< \mathcal{K}, \mathcal{H}_1, \mathcal{H}_2 >$, $\mathcal{G} =< \mathcal{K}, \mathcal{G}_1, \mathcal{G}_2 >$ respectively. Then

1. $\mathcal{H} \subseteq \mathcal{G}$ iff $\mathcal{H}_1 \subseteq \mathcal{G}_1$ and $\mathcal{H}_2 \supseteq \mathcal{G}_2$

2. $\mathcal{H} = \mathcal{G}$ iff $\mathcal{H} \subseteq \mathcal{G}$ and $\mathcal{G} \subseteq \mathcal{H}$

3. $\mathcal{H}^C =< \mathcal{K}, \mathcal{H}_2, \mathcal{H}_1 >$

4. $\tilde{\emptyset} =< \mathcal{K}, \emptyset, \mathcal{K} >, \tilde{\mathcal{K}} =< \mathcal{K}, \mathcal{K}, \emptyset >$

5. $\mathcal{H} \cup \mathcal{G} =< \mathcal{K}, \mathcal{H}_1 \cup \mathcal{G}_1, \mathcal{H}_2 \cap \mathcal{G}_2 >$

6. $\mathcal{H} \cap \mathcal{G} =< \mathcal{K}, \mathcal{H}_1 \cap \mathcal{G}_1, \mathcal{H}_2 \cup \mathcal{G}_2 >$.

Furthermore, let $\{A_\alpha / \alpha \epsilon \text{ J}\}$ be an arbitrary family of intuitionistic sets in $\mathcal{K}$, where $A_\alpha =< \mathcal{K}, \mathcal{H}_\alpha^{(1)}, \mathcal{H}_\alpha^{(2)} >$. Then

(i) $\cap \mathcal{H}_\alpha =< \mathcal{K}, \cap \mathcal{H}_\alpha^{(1)}, \cup \mathcal{H}_\alpha^{(2)} > \mathcal{K}$

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 3182-3187
https://publishoa.com
ISSN: 1309-3452

(ii) $\cup \mathcal{H}_\alpha =< \mathcal{K}, \cup \mathcal{H}_\alpha^{(1)}, \cap \mathcal{H}_\alpha^{(2)} >$

Definition 2.3 [5]: An intuitionistic topology is (for short IT) on a non-empty set $\mathcal{K}$ is a family τ of intutionistic sets in $\mathcal{K}$ satisfying following axioms.

1) $\tilde{\emptyset}, \tilde{\mathcal{K}} \in \tau$

2) $\mathcal{G}_1 \cap \mathcal{G}_2 \in \tau$, for any $\mathcal{G}_1, \mathcal{G}_2 \in \tau$

3) $\cup \mathcal{G}_\alpha \in \tau$ for any arbitrary family $\{\mathcal{G}_i : \mathcal{G}_\alpha / \alpha \epsilon \mathrm{J}\}$ where $(\mathcal{K}, \tau)$ is called an intuitionistic topological space and any intuitionistic set is called an intuitionistic open set (for short $\mathcal{I}OS$) in $\mathcal{K}$. The complement $\mathcal{H}^C$ of an $\mathcal{I}OS$ of $\mathcal{H}$ is called an intuitionistic closed set (for short $\mathcal{I}CS$) in $\mathcal{K}$.

Definition 2.4 [5]: Let $(\mathcal{K}, \tau)$ be an intuitionistic topological space and $\mathcal{H} =< \mathcal{K}, \mathcal{H}_1, \mathcal{H}_2 >$ be an IS in $\mathcal{K}$. Then the $\mathcal{I}$-interior and $\mathcal{I}$-closure of $\mathcal{H}$ are defined by

$\mathcal{I}int(\mathcal{H}) = \cup \{\mathcal{G} : \mathcal{G}$ is an $\mathcal{I}OS$ in $\mathcal{K}$ and $\mathcal{G} \subseteq \mathcal{H}\}$.

$\mathcal{I}cl(\mathcal{H}) = \cap \{\mathcal{S} : \mathcal{S}$ is an $\mathcal{I}CS$ in $\mathcal{K}$ and $\mathcal{H} \subseteq \mathcal{S}\}$

It can be shown that $\mathcal{I}cl(\mathcal{H})$ is an $\mathcal{I}CS$ and $\mathcal{I}int\ (\mathcal{H})$ is an $\mathcal{I}OS$ in $\mathcal{K}$ and $\mathcal{H}$ is an $\mathcal{I}CS$ in $\mathcal{K}$ iff $\mathcal{I}cl(\mathcal{H}) = \mathcal{H}$ and $\mathcal{H}$ is an $\mathcal{I}OS$ in $\mathcal{K}$ iff $\mathcal{I}int(\mathcal{H}) = \mathcal{H}$.

Definition 2.5 [6]: Let $(\mathcal{K}, \tau)$ be an intutionistic topological space. An intuitionistic set $\mathcal{H}$ of $\mathcal{K}$ is said to be

Intuitionistic semi-open if $\mathcal{H} \subseteq \mathcal{I}cl\big(\mathcal{I}int\ (\mathcal{H})\big)$.

Intuitionistic pre-open if $\mathcal{H} \subseteq \mathcal{I}int(\mathcal{I}cl\ (\mathcal{H}))$.

Intuitionistic α-open if $\mathcal{H} \subseteq \mathcal{I}int\Big(\mathcal{I}cl\big(\mathcal{I}int\ (\mathcal{H})\big)\Big)$.

Intuitionistic β-open if $\mathcal{H} \subseteq \mathcal{I}cl\ (\mathcal{I}int(\mathcal{I}cl\ (\mathcal{H})))$.

The family of all intuitionistic semi-open, intuitionistic pre-open, intuitionistic α-open and intuitionistic β-open sets of $(\mathcal{K}, \tau)$ are denoted by $\mathcal{I}$SOS, $\mathcal{I}$POS, $\mathcal{I}\alpha$OS, and $\mathcal{I}\beta$OS respectively.

Definition 2.6 [8]: A subset $\mathcal{M}$ of intuitionistic topological space $(\mathcal{K}, \tau)$ is called an intuitionistic w-closed set (briefly $\mathcal{I}w$ -closed) if $\mathcal{I}cl\ (\mathcal{M}) \subseteq \mathcal{F}$ whenever $\mathcal{M} \subseteq \mathcal{F}$ and $\mathcal{F}$ is intutionistic semi-open in $\mathcal{K}$.

Definition 2.7 [9]: A subset $\mathcal{M}$ of intuitionistic topological space $(\mathcal{K}, \tau)$ is called an intuitionistic generalized-closed set (briefly $\mathcal{I}g$-closed) if $\mathcal{I}cl\ (\mathcal{M}) \subseteq \mathcal{F}$ whenever $\mathcal{M} \subseteq \mathcal{F}$ and $\mathcal{F}$ is $\mathcal{I}$-open in $\mathcal{K}$.

Definition 2.8 [2]: Let $\mathcal{K}$ be a non empty set and $p \in \mathcal{K}$ a fixed element in $\mathcal{K}$. Then the intutionistic set $\tilde{p} =< \mathcal{K}, \{p\}, \{p\}^c >$ is called intutionistic point and $\tilde{\tilde{p}} =< x, \emptyset, \{p\}^c >$ is called intutionistic vanishing point.

Definition 2.9 [2]: Let $p \in \mathcal{K}$ and $\mathcal{H} =< \mathcal{K}, \mathcal{H}_1, \mathcal{H}_2 >$ be an intutionistic set. Then

(i) $\tilde{p} \subseteq \mathcal{H}$ iff $\tilde{p} \in \mathcal{H}_1$

(ii) $\tilde{\tilde{p}} \subseteq \mathcal{H}$ iff $\tilde{\tilde{p}} \in \mathcal{H}_2$

3. Intutionistic i-open Sets

Definition 3.1: An intutionistic set $\mathcal{D}$ of an Intutionistic topological space $(\mathcal{K}, \tau)$ is named as intutionistic i-open set (shortly $\mathcal{I}i$-open set) if there exist an intutionistic open set $\mathcal{H} \neq \tilde{\emptyset}$ and $\tilde{\mathcal{K}}$ such that $\mathcal{D} \subseteq \mathcal{I}cl(\mathcal{D} \cap \mathcal{H})$. The complement of $\mathcal{I}i$-open set is called $\mathcal{I}i$-closed set. The set of all intutionistic i-open sets of $(\mathcal{K}, \tau)$ is denoted by $\mathcal{I}iO$.

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 3182-3187
https://publishoa.com
ISSN: 1309-3452

Example 3.2. Let $\mathcal{K} = \{r, s, t\}$ with a family $\tau = \{\widetilde{\mathcal{K}}, \widetilde{\emptyset}, \mathcal{H}_1, \mathcal{H}_2, \mathcal{H}_3\}$ where $\mathcal{H}_1 = <\mathcal{K}, \{r\}, \{t\}>$, $\mathcal{H}_2 = <\mathcal{K}, \{r, s\}, \emptyset>$ and $\mathcal{H}_3 = <\mathcal{K}, \emptyset, \{r, t\}>$. The Intutionistic i-open sets are all the intutionistic subsets of $\mathcal{K}$.

Theorem 3.3. An intutionistic set $\mathcal{D}$ of an Intutionistic topological space $(\mathcal{K}, \tau)$ is an intutionistic i-closed set iff $\mathcal{J}int(\mathcal{D} \cup \mathcal{F}) \subseteq \mathcal{D}$ where $\mathcal{F}$ is intutionistic closed set.

Proof : Let $\mathcal{D}$ be an intutionistic i-closed set. Then, $\mathcal{D}^c = \mathcal{G}$ is intutionistic i-open. By the definition of intutionistic i-open there exists an intutionistic open set $\mathcal{H} \neq \widetilde{\emptyset}$ and $\widetilde{\mathcal{K}}$ such that $\mathcal{D}^c \subseteq \mathcal{J}cl(\mathcal{D}^c \cap \mathcal{H}) = \left(\mathcal{J}int(\mathcal{D} \cup \mathcal{H}^c)\right)^c$ which implies $\mathcal{J}int(\mathcal{D} \cup \mathcal{H}^c) \subseteq \mathcal{D}$. Let $\mathcal{H}^c = \mathcal{F}$ where $\mathcal{F}$ is intutionistic closed set. Then, $\mathcal{J}int(\mathcal{D} \cup \mathcal{F}) \subseteq \mathcal{D}$. Conversely, Let $\mathcal{F}$ be intutionistic

closed set such that $\mathcal{J}int(\mathcal{D} \cup \mathcal{F}) \subseteq \mathcal{D}$. Then $\mathcal{F}^c = \mathcal{H}$ is intutionistic open set. $\mathcal{J}int(\mathcal{D} \cup \mathcal{F}) = (\mathcal{J}cl(\mathcal{D}^c \cap \mathcal{F}^c))^c \subseteq \mathcal{D}$ which implies $\mathcal{D}^c \subseteq \mathcal{J}cl(\mathcal{D}^c \cap \mathcal{H})$ which implies $\mathcal{D}^c$ is intutionistic i-open. Hence, $\mathcal{D}$ is intutionistic i-closed.

Theorem 3.4. Each intutionistic open set is intutionistic i-open set.

Proof : Assume $\mathcal{H}$ be an intutionistic open set. Then $\mathcal{H} \subseteq \mathcal{J}cl(\mathcal{H} \cap \mathcal{H})$. Hence, $\mathcal{H}$ is an

intutionistic i-open set.

Remark 3.5. The reverse implication is not true.

Example 3.6. Let $\mathcal{K} = \{m, n\}$ with a family $\tau = \{\widetilde{\mathcal{K}}, \widetilde{\emptyset}, \mathcal{R}_1, \mathcal{R}_2\}$ where $\mathcal{R}_1 = <\mathcal{K}, \{n\}, \{m\}>$ and $\mathcal{R}_2 = <\mathcal{K}, \emptyset, \{m\}>$. $<\mathcal{K}, \emptyset, \emptyset>$ is intutionistic i-open set but, not an intutionistic open set.

Corollary 3.7. Each intutionistic closed set is intutionistic i-closed set.

Theorem 3.8. Every intutionistic regular open set is intutionistic i-open set.

Proof : Let $\mathcal{U}$ be an intutionistic regular open set. Since every intutionistic regular open is intutionistic open and by theorem 3.4., $\mathcal{U}$ is intutionistic i-open set.

Remark 3.9. The reverse implication is not true.

Example 3.10. Consider example 3.6. Here $<\mathcal{K}, \{n\}, \{m\}>$ is intutionistic i-open but not an intutionistic regular open set.

Corollary 3.11. If $\mathcal{U}$ is intutionistic regular closed set, then $\mathcal{U}$ is intutionistic i-closed set.

Theorem 3.12: Every intutionistic semi open set is intutionistic i-open set.

Proof : Take $\mathcal{B}$ be an intutionistic semi open set. Then there exists an intutionistic open set $\mathcal{G}$ such that $\mathcal{G} \subseteq \mathcal{B} \subseteq \mathcal{J}cl(\mathcal{G})$. Since $\mathcal{G} \subseteq \mathcal{B}$, $\mathcal{G} \cap \mathcal{B} = \mathcal{G}$. Therefore, $\mathcal{B} \subseteq \mathcal{J}cl(\mathcal{G} \cap \mathcal{B})$. Hence, $\mathcal{B}$ is intutionistic i-open.

Remark 3.13. The reverse implication is not true.

Example 3.14. Let $\mathcal{K} = \{\kappa, \lambda\}$ with $\tau = \{\widetilde{\mathcal{K}}, \widetilde{\emptyset}, \mathcal{V}_1, \mathcal{V}_2, \mathcal{V}_3\}$ where $\mathcal{V}_1 = <\mathcal{K}, \emptyset, \{\lambda\}>$, $V_2 = <\mathcal{K}, \{\kappa\}, \{\lambda\}>$ and $V_3 = <\mathcal{K}, \{\kappa\}, \emptyset>$. Here $<\mathcal{K}, \emptyset, \{\kappa\}>$ is intutionistic i-open set but, not an intutionistic semi-open set.

Theorem 3.15. An intutionistic set $\mathcal{S}$ is intutionistic i-open set whenever $\mathcal{S}$ is intutionistic alpha-open set.

Proof : Let $\mathcal{H}$ be an intutionistic alpha open set. Since every intutionistic alpha open set is intutionistic semi open and by theorem 3.12., $\mathcal{H}$ is intutionistic i-open set.

Remark 3.16. The reverse implication is not true.

Example 3.17. Consider example 3.14. Here, $<\mathcal{K}, \emptyset, \emptyset>$ is intutionistic i-open but not an intutionistic α-open set.

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 3182-3187
https://publishoa.com
ISSN: 1309-3452

Corollary 3.18. Every intutionistic i-closed set is intutionistic α-closed set.

Theorem 3.19. Union of two intutionistic i-open sets are intutionistic i-open set.

Proof: Let $\mathcal{G}$ and $\mathcal{H}$ be two intutionistic i-open sets. Then there exist an intutionistic open set

$\mathcal{C}$ such that $\mathcal{G} \subseteq \mathcal{I}cl(\mathcal{G} \cap \mathcal{C})$ and $\mathcal{H} \subseteq \mathcal{I}cl(\mathcal{H} \cap \mathcal{C})$. Now $\mathcal{G} \cup \mathcal{H} \subseteq \mathcal{I}cl(\mathcal{G} \cap \mathcal{C}) \cup \mathcal{I}cl(\mathcal{H} \cap \mathcal{C})$

= $\mathcal{I}cl((\mathcal{G} \cap \mathcal{C}) \cup (\mathcal{H} \cap \mathcal{C}))$= $Icl((\mathcal{G} \cup \mathcal{H}) \cap G)$.Therefore, $\mathcal{G} \cup \mathcal{H}$ is intutionistic i-open set.

Corollary 3.20. Intersection of two intutionistic i-closed sets are intutionistic i-closed set.

Proof: Let $\mathcal{F}$ and $\mathcal{L}$ be two intutionistic i-closed sets. Then, $\mathcal{F}^c$ and $\mathcal{L}^c$ are intutionistic i-

open sets. By the above theorem, $\mathcal{F}^c \cup \mathcal{L}^c = (\mathcal{F} \cap \mathcal{L})^c$ is intutionistic i-open. Hence, $\mathcal{F} \cap \mathcal{L}$ is intutionistic i-closed set.

Remark 3.21. Intersection of intutionistic i-open sets are not intutionistic i-open.

Example 3.22. Let $\mathcal{K} = \{17,19,21\}$ with $\tau = \{\widetilde{\mathcal{K}}, \widetilde{\emptyset}, \mathcal{V}_1, \mathcal{V}_2, \mathcal{V}_3\}$ where $\mathcal{V}_1 = < \mathcal{K}, \{17\}, \{19\} >$, $\mathcal{V}_2 = < \mathcal{K}, \emptyset, \{19\} >$, $\mathcal{V}_3 = < \mathcal{K}, \{17,21\}, \emptyset >$ and $\mathcal{V}_4 = < \mathcal{K}, \{17\}, \emptyset >$. Here, $< \mathcal{K}, \{19\}, \emptyset >$ and $< \mathcal{K}, \{19,21\}, \{17\} >$ are intutionistic i-open sets but, their intersection $< \mathcal{K}, \{19\}, \{17\} >$ which is not intutionistic i-open.

Remark 3.23. Union of intutionistic i-closed sets are not intutionistic i-closed sets.

Example 3.24. Consider example 3.22. $< \mathcal{K}, \{17\}, \{19,21\} >$ and $< \mathcal{K}, \emptyset, \{19\} >$ are intutionistic i-closed sets but, their union $< \mathcal{K}, \{17\}, \{19\} >$ which is not intutionistic i-closed set.

Remark 3.25. Intutionistic i-open and Intutionistic pre-open are independent.

Remark 3.26. Intutionistic i-open and Intutionistic β-open are independent.

Example 3.27. Let $\mathcal{K} = \{\zeta, \eta\}$ with $\tau = \{\widetilde{\mathcal{K}}, \widetilde{\emptyset}, \mathcal{R}_1, \mathcal{R}_2\}$ where $\mathcal{R}_1 = < \mathcal{K}, \zeta, \emptyset >$ and $\mathcal{R}_2 = < \mathcal{K}, \emptyset, \emptyset >$. Here, $< \mathcal{K}, \eta, \emptyset >$ is both intutionistic pre-open and intutionistic β-open but not an intutionistic i-open set. Also, $< \mathcal{K}, \emptyset, \zeta >$ is intutionistic i-open but not intutionistic pre-open and intutionistic β-open sets. Hence, intutionistic pre-open and intutionistic i-open , intutionistic β-open and intutionistic i-open are independent.

Remark 3.26. Intutionistic i-open and Intutionistic g-open are independent.

Remark 3.27. Intutionistic i-open and Intutionistic w-open are independent.

Example 3.28. Let $\mathcal{K} = \{\Gamma, \Delta\}$ with $\tau = \{\widetilde{\mathcal{K}}, \widetilde{\emptyset}, \mathcal{S}_1, \mathcal{S}_2\}$ where $\mathcal{S}_1 = < \mathcal{K}, \emptyset, \emptyset >$ and $\mathcal{S}_2 = < \mathcal{K}, \Delta, \emptyset >$. Here, $< \mathcal{K}, \Gamma, \emptyset >$ is both intutionistic g-open and intutionistic w-open but not an intutionistic i-open set. Also, $< \mathcal{K}, \emptyset, \Delta >$ is intutionistic i-open but not intutionistic g-open and intutionistic w-open sets. Hence, intutionistic g-open and intutionistic i-open , intutionistic w-open and intutionistic i-open are independent.

Definition 3.29. An intutionistic set $\mathcal{D}$ of an Intutionistic topological space $(\mathcal{K}, \tau)$ is named as intutionistic $i\alpha$-open set (shortly $\mathcal{I}i\alpha$-open set) if there exist an intutionistic alpha open set $\mathcal{H} \neq \widetilde{\emptyset}$ and $\widetilde{\mathcal{K}}$ such that $\mathcal{D} \subseteq \mathcal{I}cl(\mathcal{D} \cap \mathcal{H})$.

Definition 3.30. An intutionistic set $\mathcal{G}$ of an Intutionistic topological space $(\mathcal{K}, \tau)$ is called as intutionistic is-open set if there exist an intutionistic semi open set $\mathcal{M} \neq \widetilde{\emptyset}$ and $\widetilde{\mathcal{K}}$ such that $\mathcal{G} \subseteq \mathcal{I}cl(\mathcal{G} \cap \mathcal{M})$.

Definition 3.31. An intutionistic set $\mathcal{M}$ of an Intutionistic topological space $(\mathcal{K}, \tau)$ is said to

be an intutionistic ip-open set if there exist an intutionistic pre-open set $\mathcal{P} \neq \widetilde{\emptyset}$ and $\widetilde{\mathcal{K}}$ such that $\mathcal{M} \subseteq \mathcal{I}cl(\mathcal{M} \cap \mathcal{P})$.

Theorem 3.32. Every intutionistic i-open set is intutionistic $i\alpha$-open set(respectively intutionistic is-open set, intutionistic ip-open set).

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 3182-3187
https://publishoa.com
ISSN: 1309-3452

Proof : Let $\mathcal{R}$ be an intutionistic i-open set. Then there exist an intutionistic open set $\mathcal{M} \neq \widetilde{\emptyset}$ and $\widetilde{\mathcal{K}}$ such that $\mathcal{R} \subseteq \mathcal{I}cl(\mathcal{R} \cap \mathcal{M})$. Since every intutionistic open set is intutionistic alpha open set(respectively intutionistic semi open, intutionistic pre-open), $\mathcal{R}$ is intutionistic iα-open set(respectively intutionistic is-open set, intutionistic ip-open set).

Remark 3.33. The reverse implication is false.

Example 3.34. Let $\mathcal{K} = \{4,8\}$ with a family $\tau = \{\widetilde{\mathcal{K}}, \widetilde{\emptyset}, \mathcal{L}_1, \mathcal{L}_2\}$ where $\mathcal{L}_1 = <\mathcal{K}, \{8\}, \{4\}>$ and $\mathcal{L}_2 = <\mathcal{K}, \emptyset, \{4\}>$. $<\mathcal{K}, \emptyset, \{8\}>$ is intutionistic $i\alpha$-open set but, not an intutionistic i-open set.

Example 3.35. Let $\mathcal{K} = \{e, \not{f}\}$ with a family $\tau = \{\widetilde{\mathcal{K}}, \widetilde{\emptyset}, \mathcal{T}_1, \mathcal{T}_2\}$ where $\mathcal{T}_1 = <\mathcal{K}, \emptyset, \emptyset>$ and $\mathcal{T}_2 = <\mathcal{K}, \{\not{f}\}, \emptyset>$. $<\mathcal{K}, \{e\}, \emptyset>$ is intutionistic ip-open set but, not an intutionistic i-open set.

Example 3.36. Let $\mathcal{K} = \{\mathbb{k}, \mathbb{l}\}$ with a family $\tau = \{\widetilde{\mathcal{K}}, \widetilde{\emptyset}, \mathcal{S}_1, \mathcal{S}_2, \mathcal{S}_3\}$ where $\mathcal{S}_1 = <\mathcal{K}, \emptyset, \mathbb{l}>$, $\mathcal{S}_1 = <\mathcal{K}, \{\mathbb{k}\}, \{\mathbb{l}\}>$and $\mathcal{S}_3 = <\mathcal{K}, \{\mathbb{k}\}, \emptyset>$. $<\mathcal{K}, \{\mathbb{l}\}, \{\mathbb{k}\}>$ is intutionistic is-open set but, not an intutionistic i-open set.

Theorem 3.37. If A is intutionistic i-open and B is intutionistic open(respectively intutionistic α-open, intutionistic semi open, intutionistic regular open) then $A \cup B$ is intutionistic i-open.

Proof : Obvious

4. Some characterizations on Intutionistic i-open sets

Definition 4.1. Let $(\mathcal{K}, \tau)$ be an Intutionistic topological space and let $\mathcal{H} \subseteq \mathcal{K}$. The intutionistic i-interior of $\mathcal{H}$ is defined as the union of all intutionistic i-open sets contained in $\mathcal{K}$ and is denoted by $\mathcal{I}int_i(\mathcal{H})$. It is clear that $\mathcal{I}int_i(\mathcal{H})$ is the largest intutionistic i-open set, for any subset $\mathcal{H}$ of $\mathcal{K}$.

Proposition 4.2. Let $(\mathcal{K}, \tau)$ be an ITS and let $\mathcal{H} \subseteq \mathcal{G} \subseteq \mathcal{K}$. Then

1. $\mathcal{I}int_i(\mathcal{H}) \subseteq \mathcal{H}$.

2. $\mathcal{I}int_i(\mathcal{H}) \subseteq \mathcal{I}int_i(\mathcal{G})$

3. $\mathcal{H}$ is intutionistic i-open if and only if $\mathcal{H} = \mathcal{I}int_i(\mathcal{H})$

4. $\mathcal{I}int_i(\mathcal{H} \cup \mathcal{G}) = \mathcal{I}int_i(\mathcal{H}) \cup \mathcal{I}int_i(\mathcal{G})$

5. $\mathcal{I}int_i(\mathcal{H} \cap \mathcal{G}) = \mathcal{I}int_i(\mathcal{H}) \cap \mathcal{I}int_i(\mathcal{G})$

Definition 4.3. Let $(\mathcal{K}, \tau)$ be an ITS and let $\mathcal{H} \subseteq \mathcal{K}$. The intutionistic i-closure of $\mathcal{H}$ is defined

as the intersection of all intutionistic i-closed sets in $\mathcal{K}$ containing $\mathcal{H}$, and is denoted by $\mathcal{I}cl_i(\mathcal{H})$. It is clear that $\mathcal{I}cl_i(\mathcal{H})$ is the smallest intutionistic i-closed set for any subset $\mathcal{H}$ of $\mathcal{K}$.

Proposition 4.4. Let $(\mathcal{K}, \tau)$ be an ITS and let $\mathcal{H} \subseteq \mathcal{G} \subseteq \mathcal{K}$. Then

1. $\mathcal{H} \subseteq \mathcal{I}cl_i(\mathcal{H})$

2. $\mathcal{I}cl_i(\mathcal{H}) \subseteq \mathcal{I}cl_i(\mathcal{G})$

3. $\mathcal{H}$ is intutionistic i-closed if and only if $\mathcal{H} = \mathcal{I}cl_i(\mathcal{H})$

4. $\mathcal{I}cl_i(\mathcal{H} \cup \mathcal{G}) = \mathcal{I}cl_i(\mathcal{H}) \cup \mathcal{I}cl_i(\mathcal{G})$

5. $\mathcal{I}cl_i(\mathcal{H} \cap \mathcal{G}) = \mathcal{I}cl_i(\mathcal{H}) \cap \mathcal{I}cl_i(\mathcal{G})$

Proposition 4.5. Let $\mathcal{G}$ be any subset in a Intutionistic topological space $(\mathcal{K}, \tau)$, then the listed characteristics are true.

(i) $\mathcal{I}int_i(\mathcal{U} - \mathcal{G}) = \mathcal{U} - (\mathcal{I}cl_i(\mathcal{G}))$

(ii) $\mathcal{I}cl_i((\mathcal{U}-\mathcal{G})=\mathcal{U}-(\mathcal{I}int_i(\mathcal{G}))$

Proof: (i) By definition, $\mathcal{I}cl_i(\mathcal{G})=\cap\{\mathcal{B}:\mathcal{G}\subseteq\mathcal{B},\mathcal{B}$ is an intutionistic i- closed set$\}$

U – $\mathcal{I}cl_i(\mathcal{G})$ = $U\ -\cap\{\mathcal{B}:\mathcal{G}\subseteq\mathcal{B},\mathcal{B}$ is an intutionistic i- closed set$\}$ = $\cup\{\mathcal{U}-\mathcal{B}:\mathcal{G}\subseteq\mathcal{B},\mathcal{B}$ is an intutionistic i- closed set$\}$ = $\cup\{\mathcal{K}:\mathcal{K}\subseteq\mathcal{U}-\mathcal{G},\mathcal{K}$ is an intutionistic i-open set$\}$ = $\mathcal{I}int_i(\mathcal{U}-\mathcal{G})$

(ii) The proof is similar to (i)

Definition 4.6. A subset $\mathcal{Z}$ of an intutionistic topological space $(\mathcal{K},\tau)$ is called an intutionistic i-neighbourhood of a point p of $\mathcal{K}$ if there exists an intutionistic i-open set $\mathcal{H}$ containing p such that $p\in\mathcal{H}\subset\mathcal{Z}$.

Definition 4.7. Let $(\mathcal{K},\tau)$ be an ITS , $p\in\mathcal{K}$ and let $\mathcal{Z}\in\mathcal{IS}(\mathcal{K})$.

(i) $\mathcal{Z}$ is called an intutionistic i-neighbourhood of $\tilde{p}$ if there exists an intutionistic i-open set

$\mathcal{H}$ such that $\tilde{p}\in\mathcal{H}\subset\mathcal{Z}$.

(ii) $\mathcal{Z}$ is called an intutionistic i-neighbourhood of $\tilde{\tilde{p}}$ if there exists an intutionistic i-open set

$\mathcal{H}$ such that $\tilde{\tilde{p}}\in\mathcal{H}\subset\mathcal{Z}$.

We denote the set of all intutionistic i-neighborhood of $\tilde{p}$ (respectively $\tilde{\tilde{p}}$) by $N_i(\tilde{p})$(respectively $N_i(\tilde{\tilde{p}})$)

Theorem 4.8. Every intutionistic neighborhood $\mathcal{M}$ of $\tilde{p}$(respectively $\tilde{\tilde{p}}$)is an intutionistic i-neighborhood of $\tilde{p}$(respectively $\tilde{\tilde{p}}$).

Proof: Let $\mathcal{M}$ be an intutionistic neighborhood of point $p\in\mathcal{K}$. By definition of intutionistic

neighborhood, there exists an intutionistic open set $\mathcal{R}$ such that $p\in\mathcal{R}\subset\mathcal{M}$. Since every intutionistic open is intutionistic i-open, $\mathcal{M}$ is a $\mathcal{I}i$-neighborhood of p.

References

[1] K Atanassov, 1986 Intuitionistic fuzzy sets , Fuzzy Sets and Systems 20 pp 87-66
[2] D Coker 1996 A note on intuitionistic sets and intuitionistic points Turkish J. Math. 20 pp 343-351
[3] D Coker 1997 An introduction to intuitionistic fuzzy topological spaces , Fuzzy Sets And Systems 88 pp 81-89
[4] D Coker On Neighborhood Structures in Intutionistic Topological Spaces, Mathematica Balkanica, New Series, Vol. 12, 1998, pp 284-293
[5] D Coker 2000 An introduction to intuitionistic topological spaces Busefal 81 pp 51-56
[6] Y. Gnanambal On generalized preregular closed sets in topological spaces. Indian J. Pure. Appl. Math., 28(3)(1997), 351-360.
[7] A.A. Mohammed, and S.W. Askandar, (2012), "On i-open sets", UAE Math Day Conference, American Univ. of Sharjah, April 14.
[8] Taha H. Jassim Samer R. Yaseen Luma S. AbdualBaqi. Some Generalised Sets and Mappings in Intuionistic Topological Spaces., Journal of AL-Qadisiyah for computer science and mathematics Vol.7 No.2 Year 2015
[9] Younis J.Yaseen and Asmaa G. Raouf, On generalization closed set and generlized continuity on intuitionistic topological spaces, J. of al-anbar university for pure science, Vol.3, No.1 (2009)