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AUM Block Coloring for some Perfect Trees

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Abstract

Graph coloring is one of the important fields of study in graph theory, with the coloring techniques applied to heterogeneous fields, including human resource management, time management, telecommunications, etc. The new innovative coloring approach AUM block coloring introduced by the authors is being continued for perfect trees in this paper.

For perfect binary trees of level 1,2,3 AUM block chromatic number is obtained and the generalization of results upto n is also given. For perfect ternary trees of level 1,2,3 AUM block chromatic number is established. The results are generalized upto level n perfect ternary trees. Extending the results to n-ary trees in general, AUM block chromatic number for perfect n-ary trees of level 1,2,3 is computed. The generalization upto n is also presented with suitable examples, wherever necessary.

Keywords: coloring, block coloring, AUM block coloring, perfect binary trees, ternary trees.

AMS Mathematics Subject Classification (2010): 05C15, 05C69

1. Introduction

Graph coloring procedure is one of the effective tools used to solve real-world problems in heterogenous fields. Taking an insight into the history of the development, first occurrence can be traced back to an elegant 'proof' of the four-colour theorem was published by [1]. Thus initiated the theory of graph coloring. Vertex coloring was introduced by Brooks, R. in 1941 [2]. Edge coloring of the graph was introduced by Andersen, Lars Døvling in 1977 [3]. There are several coloring of graphs studied across the globe namely dominator coloring [4]–[7], Total dominator coloring [8], power dominator coloring [9]–[13], global dominator coloring [14] etc.

In 2022, A. Uma Maheswari and Bala Samuvel J introduced the new concept of coloring for graphs namely, AUM block coloring [15].

Binary trees, ternary trees and n −ary trees are the type of trees which has exactly two edges and three edges and n edges connected with the root vertex respectively. Binary trees can be classified as full binary tree, complete binary tree and perfect binary tree. Similarly, ternary trees can be classified as full ternary tree, complete ternary tree and perfect ternary tree and n-ary trees can be classified as full n-ary tree, complete n-ary tree and perfect n-ary tree.

A. Uma Maheswari and S. Azhagarasi obtained AUM block sum labelling & AUM block labelling for perfect binary trees.

Mohaimen-Bin-Noor found antimagic labelling for perfect binary tree in [16].

Let us consider finite, connected and perfect trees. Note that in perfect binary trees, perfect ternary trees and perfect n −ary trees we have three categories in terms of their vertices:

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Level $l = 0$, there is only one vertex as root and no edges and no blocks.

Level $l = 1$, there are only root and two leaves.

Level $l > 1$, there are root, leaves and other internal vertices.

The number of vertices for perfect binary tree $T_{2,l}, (l \geq 1), |V(T_{2,l})| = 2^{l+1} - 1$.

The number of edges for perfect binary tree $T_{2,l}, (l \geq 1), |E(T_{2,l})| = 2(2^l - 1)$.

The number of vertices for perfect ternary tree $T_{3,l}, (l \geq 1), |V(T_{3,l})| = \frac{3^{l+1}-1}{2}$.

The number of edges for perfect ternary tree $T_{3,l}, (l \geq 1), |E(T_{3,l})| = \frac{3^{l+1}-1}{2} - 1$.

The number of vertices of perfect n −ary tree $T_{n,l}, (l \geq 1)$, $|V(T_{n,l})| = \frac{n^{l+1}-1}{n-1}$.

The number of edges of perfect n −ary tree $T_{n,l}, (l \geq 1)$, $|V(T_{n,l})| = \frac{n^{l+1}-1}{n-1} - 1$.

The main aim of this paper is to find AUM block chromatic number for perfect binary tree, perfect ternary tree and perfect n −ary tree of level 1,2,3 up to n, with suitable examples.

2. Preliminaries

The definitions required for this paper are listed below.

Definition 1[17] **Graph Coloring:**
In a graph, coloring is an assignment of colors to the vertices or edges or both subject to certain condition(s).

Definition 2[[18]**] Block Graph:**
A graph is a block graph if every block (maximal 2-connected component) is a clique. If G is any undirected graph, the block graph of G, denoted by B(G) is a non-separable maximal subgraph of the graph. It is clear that any two blocks of a graph have at most one vertex in common.

Definition 3 [16] **Perfect Binary Tree:**
A perfect binary tree is a tree in which every internal vertex is of degree 3, the root vertex is of degree *2* and the pendent vertices are of degree 1 and have the same depth.

Definition 4 [15] **AUM Block Coloring:**
AUM block coloring of a graph G is assignment of colors to the blocks of G.

Definition 5 [15] **Proper AUM Block Coloring:**
AUM block coloring of G is proper if different colors are assigned to the blocks that have a common vertex**.**
The minimum number of colors required for proper AUM block coloring of the graph G, is called AUM block chromatic number. It is denoted as χ_{Bl}.

3. Main Results

3.1 AUM block coloring for perfect binary tree

In this section we study AUM block coloring for perfect binary tree for levels $l = 1,\ l = 2,\ l = 3, l > 3$.

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Theorem 1: For level $l = 1$, the AUM block coloring of perfect binary tree $T_{2,l}$ is 2.
Proof:
For $l = 1$, let $T_{2,l}$ be theperfect binary tree with vertices $V(T_{2,1}) = \{v_1, v_2, v_3\}$, edges are $E(T_{2,1}) = \{e_1, e_2\}$ and blocks are $B(T_{2,1}) = \{B_1, B_2\}$. The graph $T_{2,1}$ has $|V(T_{2,1})| = 3$, $|E(T_{2,1})| = 2$, $|B(T_{2,1})| = 2$. i.e., this binary tree graph has three vertices, two edges and two blocks.

Assign color $c_1$to the block B_1, color c_2 to block B_2. This coloring to the blocks is proper. The AUM block chromatic number of perfect binary tree $T_{2,l}$, $l = 1$ is 2. i.e., $\chi_{Bl}(T_{2,1}) = 2$.

In the figure 1, AUM block coloring of perfect binary tree for level 1 is shown.

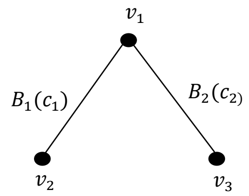

Fig.1 Perfect binary tree $T_{2,1}$

Theorem 2: For level $l = 2$, the AUM block coloring of perfect binary tree $T_{2,l}$ is 3.
Proof:
For $l = 2$, let $T_{2,l}$ be the perfect binary tree with vertices $V(T_{2,2}) = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7\}$, edges are $E(T_{2,2}) = \{e_1, e_2, e_3, e_4, e_5, e_6\}$ and blocks are $B(T_{2,2}) = \{B_1, B_2, B_3, B_4, B_5, B_6\}$. The graph $T_{2,2}$ has $|V(T_{2,2})| = 7$, $|E(T_{2,2})| = 6$, $|B(T_{2,2})| = 6$. i.e., this binary tree graph has seven vertices, six edges and six blocks.

Assign color $c_1$to the blocks B_1 & B_5, color c_2 to blocks B_2 & B_4. Colorc_3 be assigned to the blocks B_3, B_6.This coloring to the blocks is proper. The AUM block chromatic number of perfect binary tree $T_{2,l}$, $l = 2$ is 3. i.e., $\chi_{Bl}(T_{2,2}) = 3$.

In the figure 2, AUM block coloring of perfect binary tree for level 2 is shown.

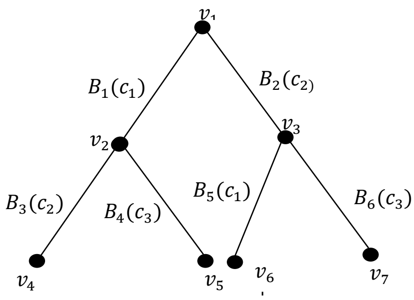

Fig. 2 Perfect binary tree $T_{2,2}$

Theorem 3: For level $l = 3$, the AUM block coloring of perfect binary tree $T_{2,l}$ is 3.
Proof:
For $l = 3$, let $T_{2,l}$ be the perfect binary tree with vertices $V(T_{2,3}) = \{v_1, v_2, v_3, v_4, \ldots, v_{15}\}$, edges are $E(T_{2,3}) = \{e_1, e_2, e_3, e_4, \ldots, e_{14}\}$ and blocks are $B(T_{2,3}) = \{B_1, B_2, B_3, B_4, \ldots, B_{14}\}$. The graph $T_{2,3}$ has $|V(T_{2,3})| = 15$, $|E(T_{2,3})| = 14$, $|B(T_{2,3})| = 14$. i.e., this binary tree graph has 15 vertices, 14 edges and 14 blocks.

Assign color $c_1$to the blocks $B_1, B_5, B_7, B_{10}, B_{14}$, color c_2 to blocks $B_2, B_4, B_8, B_{11}, B_{13}$. Color c_3 be assigned to the blocks B_3, B_6, B_9, B_{12}. This coloring to the blocks is proper. The AUM block chromatic number of perfect binary tree $T_{2,l}$, $l = 3$ is 3. i.e., $\chi_{Bl}(T_{2,3}) = 3$.

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In the figure 3, AUM block coloring of perfect binary tree for level 3 is shown.

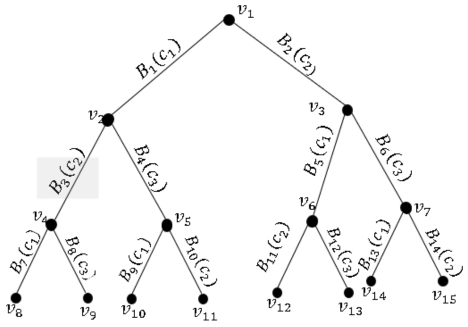

Fig.3 Perfect binary tree $T_{2,3}$

Theorem 4: For any level $l > 3$, the AUM block coloring of perfect binary tree $T_{2,l}$ is 3.
Proof:
For $l > 3$, let $T_{2,l}$ be the perfect binary tree with vertices $V(T_{2,l}) = \{v_1, v_2, v_3, \ldots, v_{2^{l+1}-1}\}$, edges are $E(T_{2,l}) = \{e_1, e_2, \ldots, e_{2(2^l-1)}\}$and blocks are $B(T_{2,l}) = \{B_1, B_2, B_3, \ldots, B_{2(2^l-1)}\}$. The graph $T_{2,l}$ has $|V(T_{2,l})| = 2^{l+1} - 1$, $|E(T_{2,l})| = 2(2^l - 1)$, $|B(T_{2,l})| = 2(2^l - 1)$. i.e., this binary tree graph has $2^{l+1} - 1$ vertices, $2(2^l - 1)$ edges and $2(2^l - 1)$ blocks.

Assign color $c_1$to the blocks $B_1, B_5, B_7, B_{10}, B_{14}$, color c_2 to blocks $B_2, B_4, B_8, B_{11}, B_{13}$. Color c_3 be assigned to the blocks B_3, B_6, B_9, B_{12}. After proper inspection of common vertex, remaining blocks are colored with either color c_1 or color c_2, or c_3. i.e., if the earlier parent block has color c_1, the children $2(2^l - 8)$ blocks are colored with remaining 3 colors c_2, c_3. This coloring to the blocks is proper. The AUM block chromatic number of perfect binary tree $T_{2,l}$, $l > 3$ is 3. i.e., $\chi_{Bl}(T_{2,l}) = 3$.

3.2 AUM block coloring for perfect ternary tree

In this section we study AUM block coloring for perfect ternary tree for levels $l = 1, l = 2, \quad l = 3, l > 3$.

Theorem 5: For level $l = 1$, the AUM block coloring of perfect ternary tree $T_{3,l}$ is 3.
Proof:
For $l = 1$, let $T_{3,l}$ be the perfect ternary tree with vertices $V(T_{3,1}) = \{v_1, v_2, v_3, v_4\}$, edges are $E(T_{3,1}) = \{e_1, e_2, e_3\}$ and blocks are $B(T_{3,1}) = \{B_1, B_2, B_3\}$. The graph $T_{3,1}$ has $|V(T_{3,1})| = 4$, $|E(T_{3,1})| = 3$, $|B(T_{3,1})| = 3$. i.e., this ternary tree graph has four vertices, three edges and three blocks.

Assign color $c_1$to the block B_1, color c_2 to block B_2, color c_3 to block B_3. This coloring to the blocks is proper. The AUM block chromatic number of perfect ternary tree $T_{3,l}$, $l = 1$ is 3. i.e., $\chi_{Bl}(T_{3,1}) = 3$.

In the figure 4, AUM block coloring of perfect ternary tree for level 1 is shown.

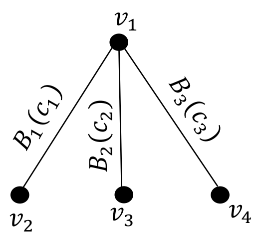

Fig.4 Perfect ternary tree $T_{3,1}$

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Theorem 6: For level $l = 2$, the AUM block coloring of perfect ternary tree $T_{3,l}$ is 4.
Proof:
For $l = 2$, let $T_{3,l}$ be the perfect ternary tree with vertices $V(T_{3,2}) = \{v_1, v_2, v_3, v_4, \ldots, v_{13}\}$, edges are $E(T_{3,2}) = \{e_1, e_2, e_3, \ldots, e_{12}\}$ and blocks are $B(T_{3,2}) = \{B_1, B_2, B_3, B_4, \ldots, B_{12}\}$. The graph $T_{3,2}$ has $|V(T_{3,2})| = 13$, $|E(T_{3,2})| = 12, |B(T_{3,2})| = 12$. i.e., this ternary tree graph has 13 vertices, 12 edges and 12 blocks.

Assign color $c_1$to the blocks B_1, B_7, B_{10}, color c_2 to blocks B_2, B_4, B_{11}. Color c_3 be assigned to the blocks B_3, B_5, B_8 color c_4 is colored to the blocks B_6, B_9, B_{12}. This coloring to the blocks is proper.

The AUM block chromatic number of perfect ternary tree $T_{3,l}$, $l = 2$ is 3. i.e., $\chi_{Bl}(T_{3,2}) = 3$.

In the figure 5, AUM block coloring of perfect ternary tree for level 2 is shown.

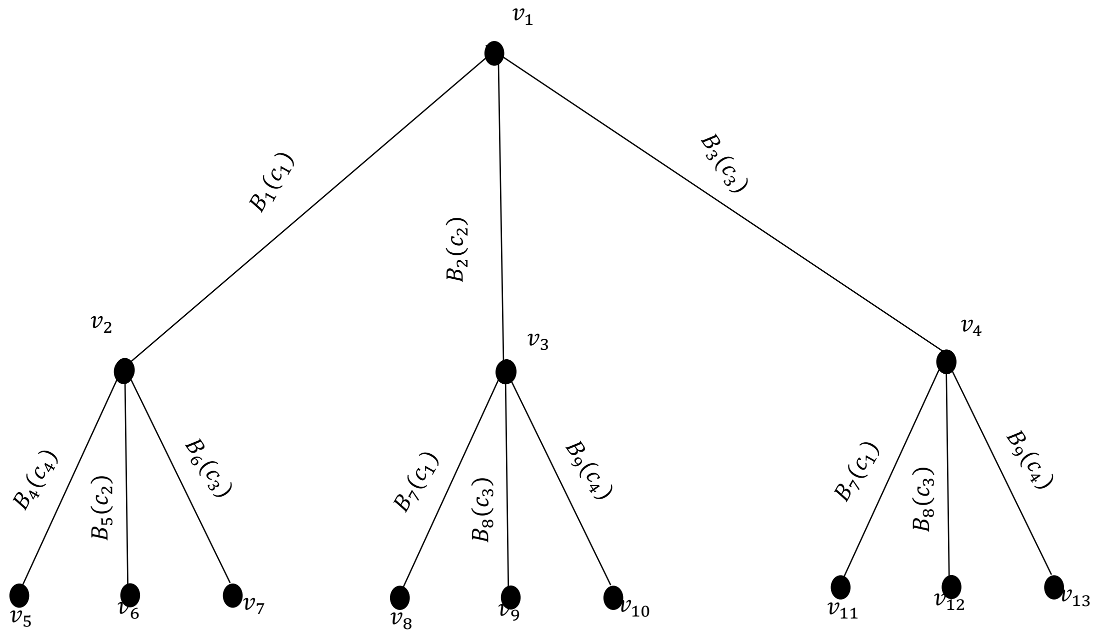

Fig.5 Perfect ternary tree $T_{3,2}$

Theorem 7: For level $l = 3$, the AUM block coloring of perfect ternary tree $T_{3,l}$ is 4.
Proof:
For $l = 3$, let $T_{3,l}$ be the perfect tree with vertices $V(T_{3,3}) = \{v_1, v_2, v_3, v_4, \ldots, v_{40}\}$, edges are $E(T_{3,3}) = \{e_1, e_2, e_3, e_4, \ldots, e_{39}\}$ and blocks are $B(T_{3,3}) = \{B_1, B_2, B_3, B_4, \ldots, B_{39}\}$. The graph$T_{3,3}$ has $|V(T_{3,3})| = 40$, $|E(T_{3,3})| = 39, |B(T_{3,3})| = 39$. i.e., this ternary tree graph has 40 vertices, 39 edges and 39 blocks.

Assign color $c_1$to the blocks $B_1, B_7, B_{10}, B_{15}, B_{17}, B_{19}, B_{26}, B_{28}, B_{34}, B_{37}$ color c_2 to blocks $B_2, B_4, B_{11}, B_{18}, B_{20}, B_{22}, B_{27}, B_{29}, B_{31}, B_{38}$. Color c_3 be assigned to the blocks $B_3, B_5, B_8, B_{13}, B_{21}, B_{23}, B_{30}, B_{32}, B_{35}, B_{39}$ color c_4 is colored to the blocks $B_6, B_9, B_{12}, B_{14}, B_{16}, B_{24}, B_{25}, B_{33}, B_{36}$. This coloring to the blocks is proper.

The AUM block chromatic number of perfect ternary tree $T_{3,l}$, $l = 3$ is 3. i.e., $\chi_{Bl}(T_{3,3}) = 4$.

In the figure 6, AUM block coloring of perfect ternary tree for level 3 is shown.

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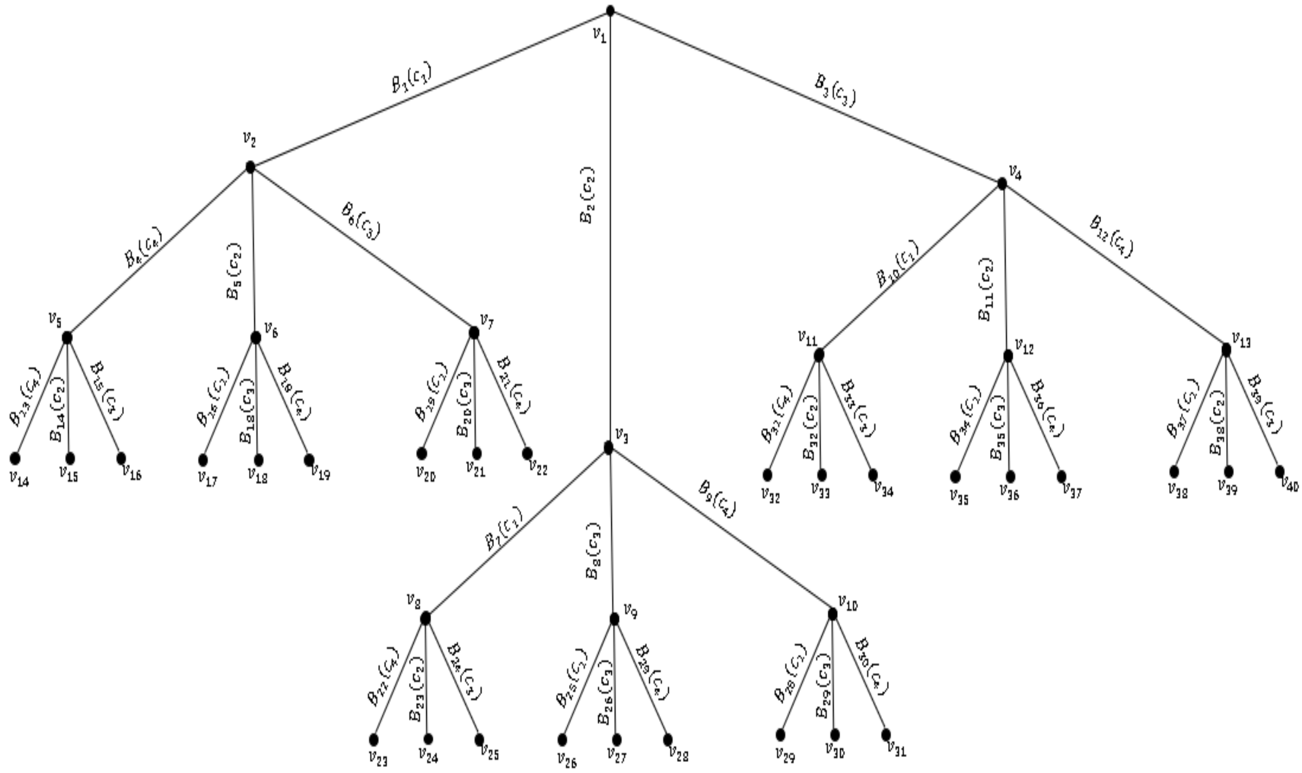

Fig.6 Perfect ternary tree $T_{3,3}$

Theorem 8: For any level $l > 3$, the AUM block coloring of perfect ternary tree $T_{3,l}$ is 4.
Proof:

For $l > 3$, let $T_{3,l}$ be the tree with vertices $V(T_{3,l}) = \left\{v_1, v_2, v_3, \dots, v_{\left(\frac{3^{l+1}-1}{2}\right)}\right\}$, edges are $E(T_{3,l}) = \left\{e_1, e_2, e_3, e_4, \dots, e_{\left(\frac{3^{l+1}-1}{2}-1\right)}\right\}$ and blocks are $B(T_{3,l}) = \left\{B_1, B_2, B_3, \dots, B_{\left(\frac{3^{l+1}-1}{2}-1\right)}\right\}$. The graph $T_{3,l}$ has $|V(T_{3,l})| = \frac{3^{l+1}-1}{2}$, $|E(T_{3,l})| = \frac{3^{l+1}-1}{2} - 1$, $|B(T_{3,l})| = \frac{3^{l+1}-1}{2} - 1$. i.e., this ternary tree graph has $\frac{3^{l+1}-1}{2}$ vertices, $\frac{3^{l+1}-1}{2} - 1$ edges and $\frac{3^{l+1}-1}{2} - 1$ blocks.

Assign color $c_1$to the blocks $B_1, B_7, B_{10}, B_{15}, B_{17}, B_{19}, B_{26}, B_{28}, B_{34}, B_{37}$ color c_2 to blocks $B_2, B_4, B_{11}, B_{18}, B_{20}, B_{22}, B_{27}, B_{29}, B_{31}, B_{38}$. Color c_3 be assigned to the blocks $B_3, B_5, B_8, B_{13}, B_{21}, B_{23}, B_{30}, B_{32}, B_{35}, B_{39}$ color c_4 is colored to the blocks $B_6, B_9, B_{12}, B_{14}, B_{16}, B_{24}, B_{25}, B_{33}, B_{36}$.

After proper inspection of common vertex, remaining blocks are colored with either color c_1 or color c_2 or color c_3 or color c_4. i.e., if the earlier parent block has color c_1, the children $\frac{3^{l+1}-1}{2} - 40$ blocks are colored with remaining 3 colors c_2, c_3, c_4.This coloring to the blocks is proper.

The AUM block chromatic number of perfect ternary tree $T_{3,l}$, $l > 3$ is 4. i.e., $\chi_{Bl}(T_{3,l}) = 4$.

3.3 AUM block coloring for perfect n -ary tree

In this section, we study AUM block coloring for perfect n −ary tree for levels $l = 1, l = 2,\ l = 3,\ l > 3$.

Theorem 9: For level $l = 1$, the AUM block coloring of perfect n -ary tree $T_{n,l}$ is n.
Proof:
For $l = 1$, let $T_{n,l}$ be the n -ary tree with vertices $V(T_{n,1}) = \{v_1, v_2, v_3, \dots, v_{n+1}\}$, edges are $E(T_{n,1}) = \{e_1, e_2, \dots, e_n\}$ and blocks are $B(T_{n,1}) = \{B_1, B_2, \dots B_n\}$. The graph $T_{n,1}$ has $|V(T_{n,1})| = n + 1$, $|E(T_{n,1})| = n, |B(T_{n,1})| = n$. i.e., this n -ary tree graph has $n + 1$ vertices, n edges and n blocks.

Assign color c_i to the block B_i, for $i = 1,2,3, \dots n$. This coloring of the blocks is proper.

The AUM block chromatic number of perfect n -ary tree $T_{n,l}$, $l = 1$ is n. i.e., $\chi_{Bl}(T_{n,1}) = n$.

Example 1: In the figure 7, AUM block coloring of perfect 4-ary tree for level 1 is shown.

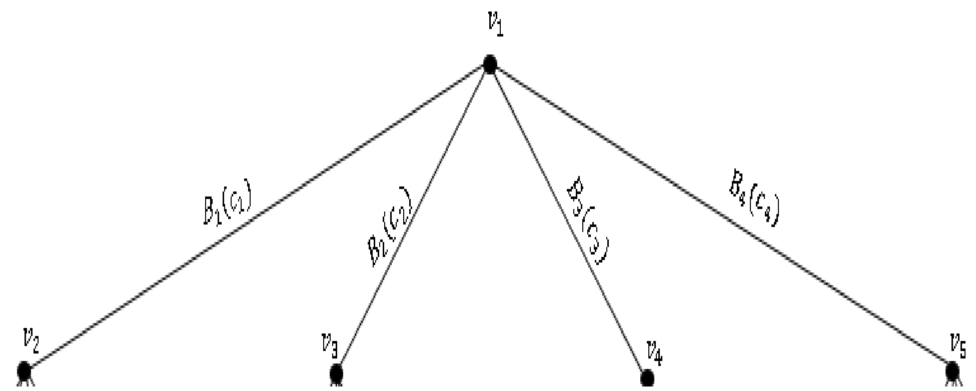

Fig: 7 Perfect 4-ary tree $T_{4,1}$

Theorem 10: For level $l = 2$, the AUM block coloring of perfect n-ary tree $T_{n,l}$ is $n + 1$.
Proof:
For $l = 2$, let $T_{n,l}$ be the perfect n-ary tree with vertices $V(T_{n,2}) = \left\{v_1, v_2, v_3, \dots, v_{\left(\frac{n^3-1}{n-1}\right)}\right\}$, edges are $E(T_{n,2}) = \left\{e_1, e_2, \dots, e_{\left(\frac{n^3-1}{n-1}-1\right)}\right\}$ and blocks are $B(T_{n,2}) = \left\{B_1, B_2, \dots, B_{\left(\frac{n^3-1}{n-1}-1\right)}\right\}$. The graph $T_{n,2}$ has $|V(T_{n,2})| = \left(\frac{n^3-1}{n-1}\right)$, $|E(T_{n,2})| = \left(\frac{n^3-1}{n-1} - 1\right), |B(T_{n,2})| = \left(\frac{n^3-1}{n-1} - 1\right)$. i.e., this n −ary tree graph has $\left(\frac{n^3-1}{n-1}\right)$ vertices, $\left(\frac{n^3-1}{n-1} - 1\right)$ edges and $\left(\frac{n^3-1}{n-1} - 1\right)$ blocks.

Assign color c_i to the blocks B_i, for $i = 1,2,3, \dots n$. After proper inspection of common vertex, remaining blocks are colored with color $c_j, 1 \le j \le n + 1$. i.e., if the earlier parent block has color c_j, the children $\left(\frac{n^3-1}{n-1} - (n + 1)\right)$ blocks are colored with remaining n colors $c_k, k \ne j,\ 1 \le k \le n + 1$. This coloring to the blocks is proper.

The AUM block chromatic number of perfect n-ary tree $T_{n,l}$, $l = 2$ is $n + 1$. i.e., $\chi_{Bl}(T_{n,2}) = n + 1$.

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Example 2: In the figure 8, AUM block coloring of perfect 4-ary tree for level 2 is shown.

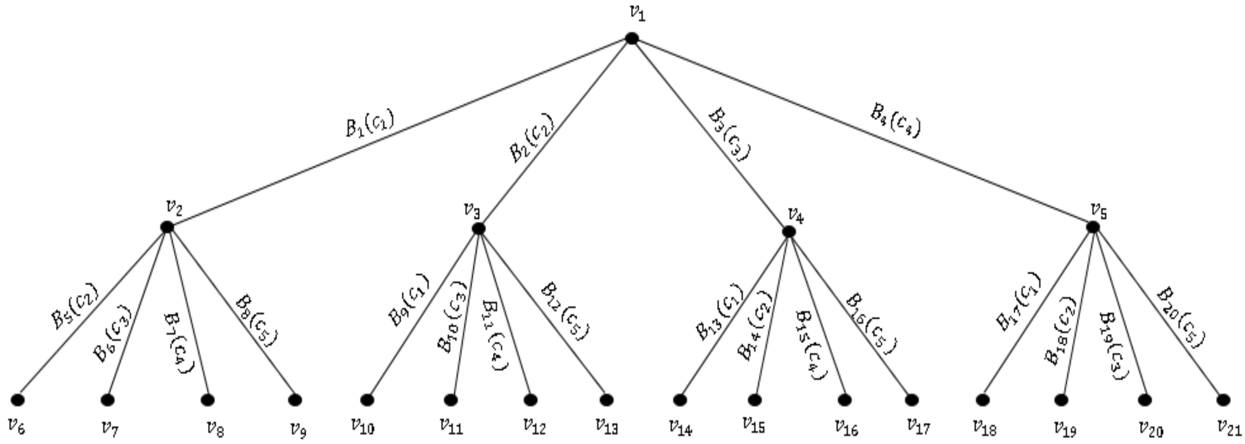

Fig: 8 Perfect 4-ary tree $T_{4,2}$

Theorem 11: For level $l = 3$, the AUM block coloring of perfect n −ary tree $T_{n,l}$ is $n + 1$.
Proof:

For $l = 3$, let $T_{n,l}$ be the perfect n-ary tree with vertices $V(T_{n,3}) = \left\{v_1, v_2, v_3, \dots, v_{\left(\frac{n^4-1}{n-1}\right)}\right\}$, edges are $E(T_{n,3}) = \left\{e_1, e_2, \dots, e_{\left(\frac{n^4-1}{n-1}-1\right)}\right\}$ and blocks are $B(T_{n,3}) = \left\{B_1, B_2, \dots, B_{\left(\frac{n^4-1}{n-1}-1\right)}\right\}$. The graph $T_{n,3}$ has $|V(T_{n,3})| = \left(\frac{n^4-1}{n-1}\right)$, $|E(T_{n,3})| = \left(\frac{n^4-1}{n-1} - 1\right)$, $|B(T_{n,3})| = \left(\frac{n^4-1}{n-1} - 1\right)$. i.e., this n −ary tree graph has $\left(\frac{n^4-1}{n-1}\right)$ vertices, $\left(\frac{n^4-1}{n-1} - 1\right)$ edges and $\left(\frac{n^4-1}{n-1} - 1\right)$ blocks.

Assign color c_i to the blocks B_i, for $i = 1,2,3, \dots n$. After proper inspection of common vertex, remaining blocks are colored with color $c_j, 1 \le j \le n + 1$. i.e., if the earlier parent block has color c_j, the children $\left(\frac{n^4-1}{n-1} - (n+1)\right)$ blocks are colored with remaining n colors $c_k, k \ne j$, $1 \le k \le n + 1$. This coloring to the blocks is proper. The AUM block chromatic number of perfect n-ary tree $T_{n,l}$, $l = 3$ is $n + 1$. i.e., $\chi_{Bl}(T_{n,3}) = n + 1$.

Theorem 12: For any level $l > 3$, the AUM block coloring of perfect n −ary tree $T_{n,l}$ is $n + 1$.
Proof:

For $l > 3$, let $T_{n,l}$ be the perfect n-ary tree with vertices $V(T_{n,l}) = \left\{v_1, v_2, v_3, \dots, v_{\left(\frac{n^{l+1}-1}{n-1}\right)}\right\}$, edges are $E(T_{n,l}) = \left\{e_1, e_2, \dots, e_{\left(\frac{n^{l+1}-1}{n-1}-1\right)}\right\}$ and blocks are $B(T_{n,l}) = \left\{B_1, B_2, \dots, B_{\left(\frac{n^{l+1}-1}{n-1}-1\right)}\right\}$. The graph $T_{n,l}$ has $|V(T_{n,l})| = \left(\frac{n^{l+1}-1}{n-1}\right)$, $|E(T_{n,l})| = \left(\frac{n^{l+1}-1}{n-1} - 1\right)$, $|B(T_{n,l})| = \left(\frac{n^{l+1}-1}{n-1} - 1\right)$. i.e., this n −ary tree graph has $\left(\frac{n^{l+1}-1}{n-1}\right)$ vertices, $\left(\frac{n^{l+1}-1}{n-1} - 1\right)$ edges and $\left(\frac{n^{l+1}-1}{n-1} - 1\right)$ blocks.

Assign color c_i to the blocks B_i, for $i = 1,2,3, \dots n.$ After proper inspection of common vertex, remaining blocks are colored with color $c_j, 1 \leq j \leq n+1$. i.e., if the earlier parent block has color c_j, the children $\left(\frac{n^{l+1}-1}{n-1} - (n+1)\right)$ blocks are colored with remaining n colors c_k, $k \neq j$. This coloring to the blocks is proper. The AUM block chromatic number of perfect n-ary tree $T_{n,l}$, $l > 3$ is $n+1$. i.e., $\chi_{Bl}(T_{n,l}) = n+1$.

4. Conclusion

The new innovative coloring approach AUM block coloring is carried out for perfect binary trees. AUM block chromatic number is established for perfect binary tree, perfect ternary trees, perfect n-ary trees with suitable examples. There is lot of scope for determining AUM block coloring and chromatic number for other graph families.

5. References

[1] A. B. Kempe, "On the Geographical Problem of the Four Colours," 1879. [Online]. Available: https://www.jstor.org/stable/2369235

[2] R. L. Brooks, "On colouring the nodes of a network," *Mathematical Proceedings of the Cambridge Philosophical Society*, vol. 37, no. 2, pp. 194–197, Apr. 1941, doi: 10.1017/S030500410002168X.

[3] Andersen and Lars DØvling, "ON EDGE-COLOURINGS OF GRAPHS," *Mathematica Scandinavica*, vol. 40, no. 2, pp. 161–175, 1977.

[4] S. Arumugam, J. Bagga, and K. R. Chandrasekar, "On dominator colorings in graphs," *Proceedings - Mathematical Sciences*, vol. 122, no. 4, pp. 561–571, Nov. 2012, doi: 10.1007/s12044-012-0092-5.

[5] S. H. Ralucca Gera, Craig Rasmussen, "Dominator Colorings and Safe Clique Partitions," *Congressus Numerantium*, 2006.

[6] R. Gera, "On the Dominator Colorings in Bipartite Graphs," in *Fourth International Conference on Information Technology (ITNG'07)*, 2007, pp. 947–952.

[7] K. Kavitha and N. G. David, "Dominator Coloring of Some Classes of Graphs," *International Journal of Mathematical Archive-3(11)*, vol. 3, no. 11, pp. 3954–3957, 2012.

[8] B. Tamilselvi and S. Kokila, "Dominator coloring and total dominator coloring of certains graphs," vol. 3, no. 12, pp. 151–154, 2017.

[9] K. S. Kumar, N. G. David, and K. G. Subramanian, "Graphs and Power Dominator Colorings," *Annals of Pure and Applied Mathematics* , vol. 11, no. 2, pp. 67–71, 2016.

[10] A. Uma Maheswari and B. Samuvel, "Power Dominator Coloring for Various Graphs," *Journal of the Maharaja Sayajirao University of Baroda*, vol. 54, no. 2(I), pp. 119–123, 2020.

[11] A. Uma Maheswari, B. Samuvel, and S. Azhagarasi, "Power Dominator Coloring of Special Kind of Graphs," *Kala Sarovar*, vol. 23, no. 04(XI), pp. 58–62, 2020.

[12] A. Uma Maheswari and B. Samuvel, "Power Dominator Coloring for Vertex Switching of some Graphs," *Compliance Engineering Journal*, vol. 13, no. 5, pp. 93–103, 2022.

[13] A. Uma Maheswari and B. Samuvel, "Power dominator chromatic number for some special graphs," *International Journal of Innovative Technology and Exploring Engineering*, vol. 8, no. 12, pp. 3957–3960, 2019, doi: 10.35940/ijitee.L3466.1081219.

[14] I. S. Hamid and M. Rajeswari, "Global dominator coloring of graphs," *Discussiones Mathematicae - Graph Theory*, vol. 39, no. 1, pp. 325–339, 2019, doi: 10.7151/dmgt.2089.

[15] A. Uma Maheswari and B. Samuvel, "New Coloring for Blocks - AUM Block Coloring for Standard Graphs," *Compliance Engineering Journal*, vol. 13, no. 5, pp. 68–77, 2022.

[16] Mohaimen-Bin-Noor and M. Manzurul Hasan, "Antimagic labelling of any perfect binary tree," Jan. 2020. doi: 10.1145/3377049.3377109.

[17] J. A. Bondy and U. S. R. Murty, *Graph Theory with Applications*, vol. 62, no. 419. 1978. doi: 10.2307/3617646.

[18] F. Harary, *Graph theory*, vol. 1. 1968.