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Q-Fuzzy Sub-Bi-Group of a Group and Level Q-Fuzzy Sub-Bi-Group

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ABSTRACT

In this paper, we introduce the concept of Q-fuzzy sub-bi-group and t-level Q-fuzzy sub-bi-group of the bi-group and discuss some main results.

Keywords:-fuzzy subgroup, Q-fuzzy set, Q-fuzzy subgroup, fuzzy sub-bi-group and Q-fuzzy sub-bi-group.

1. INTRODUCTION

The fuzzy subset was mathematically formulated by Zadeh L.A. Then it has applied in real life problem, because every object encountered in this real physical world carries some degree of fuzziness. The concept of fuzzy subgroup was introduced by Rosenfeld [5]. The construction of new structure of Q-fuzzy subgroup given by Solairaju.A and Nagarajan.R [6].Vasantha Kandasamy W.B [7] was develop the concept of fuzzy sub-bi-group. In this paper,

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we introduce the concept of Q-fuzzy sub-bi-group and some main results.

2. PRELIMINARIES

2.1 Definition [6]

A Q- fuzzy set μ is called Q-fuzzy subgroup of a group G if for all $x, y \in G$, and $q \in Q$

i) $\mu(xy, q) \geq \min\{\mu(x, q), \mu(y, q)\}$

ii) $\mu(x^{-1}, q) = \mu(x, q)$

2.2Definition [7]

Let $G = (G_1 \cup G_2, +, \cdot)$ be a bi-group. Then $\mu: G \to [0, 1]$ is said to be a fuzzy sub-bi-group of the bi-group G if there exist two fuzzy subsets μ_1(of G_1) and μ_2(of G_2) such that

(i) $(\mu_1, +)$ is a fuzzy subgroup of $(G_1, +)$

(ii) $(\mu_2, \cdot)$ is a fuzzy subgroup of $(G_2, \cdot)$ and

(iii) $\mu = \mu_1 \cup \mu_2$

3. MAIN RESULTS OF Q-FUZZY SUB-bi-GROUP AND LEVEL Q-FUZZY SUB-bi-GROUP

3.1 Definition

Let μ_1 be a Q-fuzzy subset of a set X₁ and μ_2 be a Q-fuzzy subset of a set X_2, then the fuzzy union of two Q-fuzzy sets μ_1 and μ_2 is defined as a function $\mu_1 \cup \mu_2: (X_1 \cup X_2) \times Q \to [0, 1]$ given by $(\mu_1 \cup \mu_2)(x, q) =$

$$\begin{cases} Max\,\{\mu_1(x, q), \mu_2(x, q)\} & if\ x \in X_1 \cap X_2 \\ \mu_1(x, q) & if\ \ x \in X_1\ and\ x \notin X_2 \\ \mu_2(x, q) & if\ \ x \in X_2\ and\ x \notin X_1 \end{cases}$$

For all $q \in Q$

3.2Definition

Let $G = (G_1 \cup G_2, +, \cdot)$ be a bi-group. Then $\mu: G \times Q \to [0, 1]$ is said to be a Q- fuzzy sub-bi-group of the bi-group G if there exist two Q-fuzzy subsets μ_1(of G_1) and μ_2(of G_2) such that

(i) $(\mu_1, +)$ is a Q- fuzzy subgroup of $(G_1, +)$

(ii) $(\mu_2, \cdot)$ is a Q-fuzzy subgroup of $(G_2, \cdot)$ and

(iii) $\mu = \mu_1 \cup \mu_2$

3.3Example

Let $G = \{0, \pm i, \pm 1, \pm 2, ..\}$ be a bi-group under the binary operation '+' and '·' where $G_1 = \{0, \pm 1, \pm 2, ...\}$ and $G_2 = \{\pm i, \pm 1\}$

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Now define $\mu: G \times Q \to [0,1]$ by $\mu(x,q) = \begin{cases} 0.5 \ \ if\ x = i, -i \\ 1 \ \ if\ x = 0,\ \pm 2,\ \pm 4, \dots \\ 0.7 \ \ if\ x = \ \pm 1, \pm 3, \dots \end{cases}$

For all $q \in Q$

Define $\mu_1: G_1 \times Q \to [0,1]$

$$\mu_1(x,q) = \begin{cases} 1 \ \ if\ x \in \{0, \pm 2, \pm 4, \dots \\ 0.7 \ \ if\ x \ \in \{\pm 1, \pm 3, \pm 5, \dots \end{cases}$$

For all $q \in Q$

$$and$$

$$\mu_2(x,q) = \begin{cases} 0.7 \ \ if\ x = 1, -1 \\ 0.5 \ \ if\ x = i, -i \end{cases}$$

For all $q \in Q$

3.4Definition

Let μ be a Q-fuzzy sub-bi-group of the bi-group G for $t \in [0,1]$. Then$G^t_{\mu=\mu_1 \cup \mu_2} = \{x \in G\ and\ q \in Q / \mu(x,q) \geq t\}$is called t-level subset of Q-fuzzy sub-bi-group μ.

3.5Definition

Let $G = G_1 \cup G_2$ be a bi-group and $\mu = \mu_1 \cup \mu_2$ be a Q-fuzzy sub-bi-group of the bi-group G. The bi-level subset of the Q-fuzzy sub-bi-group μ of the bi-group G is defined as $G^t_\mu = {G_1}^t_{\mu_1} \cup {G_2}^t_{\mu_2}$ for every $t \in [0, \min\{\mu_1(e_1,q), \mu_2(e_2,q)\}]$, where$q \in Q$ and e_1 and e_2 denote the identity elements of groups$(G_1,+)$ and $(G_2,\cdot)$ respectively.

3.6Theorem

Every bi-level subset of a Q-fuzzy sub-bi-group of a bi-group G is a sub-bi-group of the bi-group G.

Proof

Let $\mu = \mu_1 \cup \mu_2$ be the Q-fuzzy sub-bi-group of bi-group $G = (G_1 \cup G_2, +, \cdot)$

Consider the bi-level subset G^t_μ of the fuzzy sub-bi-group μ for every $t \in [0, \min\{\mu_1(e_1,q), \mu_2(e_2,q)\}]$, where for all $q \in Q$ and e_1 and e_2 denote the identity elements of groups G_1 and G_2 respectively.

Then $G^t_\mu = {G_1}^t_{\mu_1} \cup {G_2}^t_{\mu_2}$ where ${G_1}^t_{\mu_1}$ and ${G_2}^t_{\mu_2}$ are subgroups of G_1 and G_2 respectively.

(Since ${G_1}^t_{\mu_1}$ is a t-level subset of the group G_1 and ${G_2}^t_{\mu_2}$ is a t-level subset of the group G_2)

Hence by the definition of sub-bi-group G^t_μ is a sub-bi-group of the bi-group $(G,+,\cdot)$.

3.7Example

$G = \{0, \pm i, \pm 1\}$is a bi-group with respect to addition modulo 2 and multiplication.

Clearly $G_1 = \{0, 1\}$ and $G_2 = \{\pm 1, \pm i\}$are group with respect to addition modulo 2 and multiplication respectively. Define $\mu: G \times Q \to [0, 1]$ by $\mu(x, q) = \begin{cases} 1 & if\ x = 0 \\ 0.6 & if\ x = \pm 1 \\ 0.4 & if\ x = \pm i \end{cases}$ for all $q \in Q$

It is easy to prove that μ be a Q-fuzzy sub-bi-group of the bi-group G, there exist two Q-fuzzy subgroups $\mu_1: G \times Q \to [0, 1]$ and $\mu_2: G \times Q \to [0, 1]$ such that $\mu = \mu_1 \cup \mu_2$

Where

$\mu_1(x, q) = \begin{cases} 1 & if\ x = 0 \\ 0.5 & if\ x = 1 \end{cases}$ for all $q \in Q$

and

$\mu_2(x, q) = \begin{cases} 0.6 & if\ x = \pm 1 \\ 0.4 & if\ x = \pm i \end{cases}$ for all $q \in Q$

Now we calculate the bi-level subset G_μ^t for t=0.6

$$G_\mu^t = {G_1}_{\mu_1}^t \cup {G_2}_{\mu_2}^t$$

$$= \{x \in G_1 and\ q \in Q | \mu_1(x, q) \geq t\} \cup \{x \in G_2 and\ q \in Q | \mu_2(x, q) \geq t\}$$

$$= \{0\} \cup \{\pm 1\}$$

$$= \{0, \pm 1\}$$

That is $G_\mu^t = \{0, \pm 1\}$

It is easily prove that G_μ^t is a sub-bi-group of the bi-group G.

4. CONCLUSION

In this paper, we gave some important results of Q-fuzzy sub-bi-group and level Q-fuzzy sub-bi-group. This paper used to further research in Q-fuzzy subgroup.

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