JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 3200-3205
https://publishoa.com
ISSN: 1309-3452

Some Properties of $\mathcal{N}$ $\mathcal{D}_{\propto}$- Continuous

Rdab T. Abdalla [1,*] , Luma S. Abdalbaqi [2]

[1, 2] Department of Mathematics, College of Education for women, Tikrit University, Iraq.

Rdab.Abdalla@st.tu.edu.iq ; lumahhany1977@tu.edu.iq

Received 2022 March 15; **Revised** 2022 April 20; **Accepted** 2022 May 10

Abstract.

We will review in this work , a new kind of sets called $\mathcal{N}$ $\mathcal{D} \propto -$continuous are introduced and studied in $\mathcal{N}$Topological space. The class of all $\mathcal{N}$ $\mathcal{D} \propto -$open(closed) sets is restricted to the class of all $\mathcal{N} -$ continuous and $\mathcal{N}$ g-open. Also we study topological properties of $\mathcal{N}$ $-$continuous, $\mathcal{N}$ $\mathcal{D} \propto -$ homeomorp, $\mathcal{N}$ contra $\mathcal{D} \propto$ -continuous, $\mathcal{N}$slightly $\mathcal{D} \propto$ continuous and $\mathcal{N}$irresolute $\mathcal{D} \propto -$continuous.

Keywords

$\mathcal{N}$ Continuous, $\mathcal{N} \propto -$Continuous, $\mathcal{N}$ $\mathcal{D} \propto$ -Continuous.

1. Introduction and Basic Concepts

In this work we define $\mathcal{N}$ continuous functions, and $\mathcal{N}$ homeomorphisms between $\mathcal{N}$ topological spaces and derive their equivalent characterizations. $\mathcal{N}$ continuous functions have a wide range of uses, including plant growth over time, depreciation of machine and temperature at various times of the day. We have also provided a real-life example of $\mathcal{N}$ homeomorphism. the basic concepts of $\mathcal{N}$ topological spaces, $\mathcal{N}\mathcal{D} \propto$ open (closed) function, $\mathcal{N}$ $\mathcal{D} \propto$- continuous and their properties. In [1] $\mathcal{N}$ continuous if $f^{-1}(\aleph)$ is $\aleph$ open in $\mathcal{U}$ $\forall\aleph$ open in $\mathcal{V}$. In [2] defined $\mathcal{N}$ $\propto$-continuous If $f^{-1}(\aleph)$ is $\aleph \propto$ open set in $\mathcal{U}$ for any $\aleph \propto$ open in $\mathcal{V}$. In [3] $\mathcal{N}$ pre-continuous If $f^{-1}(\aleph)$ is $\aleph$ pre-open in $\mathcal{U}$ $\forall\aleph$ open $\mathcal{V}$. Proceeding from the important concepts, and their importance in topology, we will define new types of continuity as $\mathcal{N}$ $\mathcal{D} \propto -$ homeomorp, $\mathcal{N}$ contra $\mathcal{D} \propto$ -continuous, $\mathcal{N}$slightly $\mathcal{D} \propto$ continuous and $\mathcal{N}$irresolute $\mathcal{D} \propto -$continuous and prove them some important basic theories as well as review some examples in our research. For more information on this topic, see the papers [4, 5, 6]. We will symbolize the word nano with the symbol $\mathcal{N}$. Our work in this paper can be applied in papers [7, 8, 9] this is due to the importance of the subject and its applications in all areas of mathematics. Where we divided our work into three sections and gave the relationship between them.

2. Basic Theorems

Definition:2.1

Let $(\mathcal{U}, \mathcal{T}_R(\mathrm{X}))$ and $(\mathcal{V}, \mathcal{T}_{R'}(\mathrm{Y}))$ be $\mathcal{N}$ topological. Said to be $f:(\mathcal{U}, \mathcal{T}_R(\mathrm{X})) \rightarrow (\mathcal{V}, \mathcal{T}_{R'}(\mathrm{Y}))$ $\mathcal{N}$ $D \propto$ continuous if $f^{-1}(\aleph), \forall\, \aleph$– open in $\mathcal{V}$ is $\mathcal{N}$- $D \propto$ open $\aleph$ in $\mathcal{U}$.

Example:2.2

Let $\mathcal{U} = \{u, q, m, k\}$ with $\mathcal{U} / R = \{\{u\}, \{q, k\}, \{m\}\}$, let $X = \{\mathrm{u}, q, m\}$, a subset of $\mathcal{U}$. Then the $\mathcal{N}$ topology on $\mathcal{U}$ is given by $\mathcal{T}_R(X)$=$\{\mathcal{U}, \emptyset, \{\mathrm{u, m}\}, \{\mathrm{q, k}\}\}$. Let $\mathcal{V} = \{a, b, c, d\}$ with $\mathcal{V} \setminus \mathrm{R}'$=$\{\{a\}, \{b, d\}, \{c\}\}$, let $Y = \{a, b, c\}$. Then $\mathcal{T}_{R'}(\mathrm{Y})$= $\{\mathcal{V}, \emptyset, \{a, c\}, \{b, d\}\}$. Then $\mathcal{T}_{R'}^{c} = \{\emptyset, \mathcal{V}, \{b, d\}, \{a, c\}\}$, $\mathcal{T}_R^{C}(X)$ =$\{\emptyset, \mathcal{U}, \{\mathrm{q, k}\}, \{\mathrm{u, m}\}\}$,and

$\mathcal{N}$ $\propto$-open (X)=$\{\mathcal{U}, \emptyset, \{\mathrm{u, m}\}, \{\mathrm{q, k}\}\}$,and $\mathcal{N}$ $\propto$-open$(Y) = \{\mathcal{V}, \emptyset, \{a, c\}, \{b, d\}\}$, $\mathcal{N}$ $pre(X) = p(x)$,and $\mathcal{N}$ $pre(Y) = p(y)$.

$\mathcal{N}$semi(X)=$\{\mathcal{U}, \emptyset, \{\mathrm{u, m}\}, \{\mathrm{q, k}\}\}$, $\mathcal{N}semi(Y)$=$\{\mathcal{V}, \emptyset, \{a, c\}, \{b, d\}\}$,and

$\mathcal{N}$ g-closed$(X) = \{\emptyset, \mathcal{U}, \{\mathrm{u}\}, \{q\}, \{m\}, \{k\}, \{\mathrm{u}, q\}, \{\mathrm{u}, m\}, \{\mathrm{u}, k\}, \{q, m\}, \{q, k\}, \{m, k\}, \{\mathrm{u}, q, m\}, \{\mathrm{u}, q, k\},$

$\{q, m, k\}, \{\mathrm{u}, m, k\}\}$, $\mathcal{N}g$-open (X) is compiled of $\mathcal{N}g$-closed(X).

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 3200-3205
https://publishoa.com
ISSN: 1309-3452

$\mathcal{T}_R^{D\propto} O$(X)={∅,$\mathcal{U}$,{ɯ},$\{q\},\{m\},\{k\}$,{ɯ, q},{ɯ, m},{ɯ, k},$\{q,m\}$,$\{q,k\}$,$\{m,k\}$,{ɯ, q, m},{ɯ, q, k},

$\{q,m,k\}$,{ɯ, m, k}}, and $\mathcal{T}_{R'}(Y) = \{\mathcal{V},\emptyset,\{a,c\},\{b,d\}\}$. Define $f:(\mathcal{U},\mathcal{T}_R(\mathcal{U})) \to (\mathcal{V},\mathcal{T}_{R'}(Y))$ as $f(u) = a, f(q) = b, f(m) = c, f(k) = d$. Then $f^{-1}(\mathcal{V})= \mathcal{U}$, $f^{-1}(\emptyset)= \emptyset$, $f^{-1}(\{a,c\})$ = {ɯ, m} $\in \mathcal{T}_R(X)$, and $f^{-1}(\{b,d\}) = \{q,k\} \in \mathcal{T}_R(\mathcal{U})$.

∴ f is $\mathcal{N}$ continuous. Then $f^{-1}(\mathcal{V})= \mathcal{U}$, $f^{-1}(\emptyset)= \emptyset$, $f^{-1}(\{a,c\}) = \{u,m\} \in \mathcal{N}$ ∝-open(X), and $f^{-1}(\{b,d\}) = \{q,k\} \in \mathcal{N}$ ∝-open(X). So f is $\mathcal{N} \propto$ continuous. Then $f^{-1}(\mathcal{V})= \mathcal{U}$, $f^{-1}(\emptyset)= \emptyset$, $f^{-1}(\{a,c\})$ = {ɯ, m} $\in \mathcal{N}$semi(X) , and $f^{-1}(\{b,d\}) = \{q,k\} \in \mathcal{N}semi(X)$. Then f is $\mathcal{N}$ semi-continuous. Then

$f^{-1}(p(y)) = p(x) \in \mathcal{N}\, pre(\mathcal{U})$. ∴ f is $\mathcal{N}$ pre-continuous. Then

$f^{-1}(\mathcal{V})= \mathcal{U}$, $f^{-1}(\emptyset)= \emptyset$, $f^{-1}(\{a,c\})$ = {ɯ, m} $\in \mathcal{T}_R^{D\propto} O(X)$, $f^{-1}(\{b,d\}) = \{q,k\} \in \mathcal{T}_R^{D\propto} O(X)$. So f is $\mathcal{N}\ D \propto$ continuous. The theorem describes $\mathcal{N}\ D \propto$ continuous.

Theorem:2.3

let $f:(\mathcal{U},\mathcal{T}_R$(X)) → $(\mathcal{V},\mathcal{T}_{R'}$(Y)) is $\mathcal{N}\ D \propto$ continuous iff ∀ ℵ $\mathcal{N}$ closed on $\mathcal{V}$ then f^{-1}(ℵ)is $\mathcal{N}\ D \propto$ closed in $\mathcal{U}$.

Proof: It easy

We construct a characterization of $\mathcal{N}\ D \propto$ continuous function in terms of $\mathcal{N}$ D∝ closure in the following theorem.

Theorem: 2.4

Let $f:(\mathcal{U},\mathcal{T}_R$(X)) → $(\mathcal{V},\mathcal{T}_{R'}(Y))$ is $\mathcal{N}\ D \propto$ continuous iff $f(\mathcal{N}\ D \propto cl(\aleph)) \subseteq \mathcal{N}\ D \propto\ cl(f(\aleph))$ for any ℵ of $\mathcal{U}$.

Proof: Let f a $\mathcal{N}\ D \propto$ continuous and $\aleph \subseteq \mathcal{U}$. So $f(\aleph) \subseteq \mathcal{V}$. And let f is $\mathcal{N}\ D \propto$ continuous and $\mathcal{N}$ D∝ $cl(f(\aleph))$ is $\mathcal{N}$ - closed in $\mathcal{V}$, f^{-1} ($\mathcal{N}$ D∝ $cl(f(\aleph))$) is $\mathcal{N}$ - $D \propto$ closed in $\mathcal{U}$. Since $f(\aleph) \subseteq \mathcal{N}$ D∝ $cl(f(\aleph))$, $\aleph \subseteq f^{-1}$ ($\mathcal{N}\ D \propto cl(f(\aleph))$). Thus f^{-1} ($\mathcal{N}\ D \propto\ cl(f(\aleph))$) is a $\mathcal{N}$ - $D \propto$ closed ,containing ℵ. But , $\mathcal{N}\ D \propto cl(\aleph)$ is smallest $\mathcal{N}$ - $D \propto$ closed containing ℵ. Therefore $\mathcal{N}\ D \propto$ cl(ℵ) $\subseteq f^{-1}$ ($\mathcal{N}\ D \propto\ cl((\aleph))$. That is $f(\mathcal{N}\ D \propto$ cl(ℵ)) $\subseteq \mathcal{N}\ D \propto\ cl(f(\aleph))$. Conversely , let $f(\mathcal{N}\ D \propto cl(\aleph)) \subseteq \mathcal{N}\ D \propto\ cl(f(\aleph))$, ∀ℵ of $\mathcal{U}$. If F is $\mathcal{N}$ - closed in $\mathcal{V}$, since $f^{-1}(F) \subseteq \mathcal{U}$, $f(\mathcal{N}\ D \propto cl(f^{-1}$(F))) $\subseteq \mathcal{N}\ D \propto cl(f\ (f^{-1}$(F))) $= \mathcal{N}\ D \propto cl(F)$. That is , $\mathcal{N}\ D \propto cl(f^{-1}(F)) \subseteq f^{-1}$ ($\mathcal{N}\ D \propto\ cl(F)) = f^{-1}(F)$, since F is $\mathcal{N}$ - $D \propto$ closed. But $f^{-1}(F) \subseteq \mathcal{N}\ D \propto cl(f^{-1}(F))$. Then , $\mathcal{N}\ D \propto cl(f^{-1}(F) = f^{-1}(F)$. So, $f^{-1}(F)$ is $\mathcal{N}\ D \propto$ closed in $\mathcal{U}$, ∀ $\mathcal{N}$ - closed F in $\mathcal{V}$.

Remark:2.5

If $f:(\mathcal{U},\mathcal{T}_R$((X)) → $(\mathcal{V},\mathcal{T}_{R'}$(Y)) is $\mathcal{N}\ D \propto$ continuous, So $f(\mathcal{N}\ D \propto$ cl(ℵ)) is not always comparable $\mathcal{N}\ cl(f(\aleph))$ where $\aleph \subseteq \mathcal{U}$.

Example: 2.6

Let $\mathcal{U}$ ={ĸ, q, £ , ₰ }; $\mathcal{U}$ / R = {{ĸ},{ q, ₰},{£}}. Let X = {ĸ, q, £}. Then $\mathcal{T}_R(\mathcal{U})$= {$\mathcal{U}$, ∅, {ĸ, £}, {$q$, ₰}}. $\mathcal{T}_R^C(X)$ ={∅,$\mathcal{U}$,{q, ₰}, {ĸ, £}}. Let $\mathcal{V} == \{t,y,i,n\}$ with $\mathcal{V} \setminus R'$={{t,i},{y},{n}} and $Y = \{t,y\}$. Then $\mathcal{T}_{R'}(Y)$ = {$\mathcal{V}$,∅,{y},{t,y,i},{t,i}}. $\mathcal{T}_{R'}^C$(Y)={∅, $\mathcal{V}$,{t,i,n},{n},{y,n}},

$\mathcal{N}g$-closed(X) ={∅, $\mathcal{U}$, {ĸ}, {q}, {£}, {₰}, {ĸ, q}, {q, £, ₰}, {ĸ, £}, {ĸ, ₰}, {q, £}, {q, ₰}, {£, ₰}, {ĸ, q, £}, {ĸ, q, ₰},

{ĸ, £, ₰}}, $\mathcal{N}g$-open(X) is compiled of $\mathcal{N}g$-closed(X).

$\mathcal{T}_R^{D\propto} O(\mathcal{U})$={∅,$\mathcal{U}$, {ĸ}, {$q$}, {£, ₰}, {ĸ, q, £}, {ĸ, q, ₰}, {£}, {₰}, {ĸ, q}, {ĸ, £}, {ĸ, ₰}, {q, £}, {q, ₰}, {q, £, ₰},

{ĸ, £, ₰}}. Let $f:(\mathcal{U},\mathcal{T}_R$(X)) → $(\mathcal{V},\mathcal{T}_{R'}(Y))$ be given by f(ĸ) = y, $f(q) = t$, f(£) = y, f(₰) = t. Then $f^{-1}(\mathcal{V})$=$\mathcal{U}$, $f^{-1}(\emptyset)$=∅,$f^{-1}(\{y\})$={ĸ, £}, $f^{-1}(\{t,y,i\})$= $\mathcal{U}$, and $f^{-1}(\{t,i\})$={q, ₰}. That is , ∀ℵ, $\mathcal{N}$ open set in $\mathcal{V}$ then f^{-1}(ℵ)is $\mathcal{N}$ D∝ open of $\mathcal{U}$. So , f is $\mathcal{N}\ D \propto$ continuous of $\mathcal{U}$. Let A = {ĸ, £} $\subseteq \mathcal{U}$ .Then $f(\mathcal{N}$ D∝ $cl(A))$ = f({ĸ, £}) = {y}. But, $\mathcal{N}\ cl(f(\aleph))$= $\mathcal{N}\ cl(\{y\})$={y,n}.Thus , $f(\mathcal{N}\ D \propto cl(A)) \neq N\ cl(f$(A)) , even though f is $\mathcal{N}$ D∝ continuous. i.e. , when f is N continuous, The previous theorem is incorrect. The previous theorem does not hold when f is $\mathcal{N}$ continuous.

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 3200-3205
https://publishoa.com
ISSN: 1309-3452

Theorem: 2.7

Let $f:(\mathcal{U},\mathcal{T}_R(X)) \to (\mathcal{V},\mathcal{T}_{R'}(Y))$ is $\mathcal{N}D \propto$continuous on $\mathcal{U}$ iff $f^{-1}(\mathcal{N}D \propto \text{int}(B)) \subseteq \mathcal{N}D \propto int(f^{-1}(B)), \forall B \subseteq \mathcal{V}$.

Proposition: 2.8

If $f:(\mathcal{U},\mathcal{T}_R(X)) \to (\mathcal{V},\mathcal{T}_{R'}(Y))$ $\mathcal{N}$ topological spaces , $X \subseteq \mathcal{U}$, $Y \subseteq \mathcal{V}$. So for any $f:\mathcal{U} \to \mathcal{V}$, The following comparable:

(i) f is $\mathcal{N}$ $D \propto$ continuous .
(ii) $f^{-1}(\aleph)$ for any $\aleph$– closed in $\mathcal{V}$ is $\mathcal{N}$- $D \propto$ closed $\aleph$ in $\mathcal{U}$.
(iii) $f(\mathcal{N} D \propto \text{cl}(\aleph)) \subset \mathcal{N} D \propto \text{cl}(f(\aleph))$ for every subset $\aleph$ of $\mathcal{V}$.
(iv) f^{-1} of every member of the basis $\mathcal{B}_{R'}$ of $\mathcal{T}_{R'}(Y)$ is $\mathcal{N}$ $D \propto$ open in $\mathcal{U}$.
(v) $\mathcal{N} D \propto \text{cl}(f^{-1}(B)) \subseteq f^{-1}(\mathcal{N} D \propto \text{cl}(B))$, $\forall B \subseteq \mathcal{V}$.
(vi) $f^{-1}(\mathcal{N} D \propto \text{int}(B)) \subset \mathcal{N} D \propto \text{int}(f^{-1}(B))$, $\forall B \subseteq \mathcal{V}$.

Theorem: 2.9

Let $f:(\mathcal{U},\mathcal{T}_R(X)) \to (\mathcal{V},\mathcal{T}_{R'}(Y))$ is $\mathcal{N}$ $D \propto$ closed iff $\mathcal{N} D \propto \text{cl}(f(\aleph)) \subseteq f(\mathcal{N} D \propto cl(\aleph))$, $\forall \aleph \subseteq \mathcal{U}$.

Proof: If f is $\mathcal{N}$ $D \propto$ closed , so $f(\mathcal{N} D \propto cl(\aleph))$ is $\mathcal{N}$ closed, let $\mathcal{N} D \propto cl(\aleph))$ is $\mathcal{N}$ $D \propto$ closed in $\mathcal{U}$. Since $\aleph \subseteq \mathcal{N} D \propto cl(\aleph))$, $f(\aleph) \subseteq f(\mathcal{N} D \propto cl(\aleph))$. Thus $f(\mathcal{N} D \propto cl(\aleph))$ is a $\mathcal{N}$ $D \propto$ closed set containing $f(\aleph)$. Therefore , $\mathcal{N} D \propto cl(f(\aleph)) \subseteq f(\mathcal{N} D \propto cl(\aleph))$. Conversely , if $\mathcal{N} D \propto cl(\aleph)) \subseteq f(\mathcal{N} D \propto cl(\aleph))$, $\forall \aleph \subseteq \mathcal{U}$ and if F is $\mathcal{N}$ $D \propto$ closed, so $\mathcal{N} D \propto cl(F) = F$ and $f(F) \subseteq \mathcal{N} D \propto cl(f(F)) \subseteq f(\mathcal{N} D \propto cl(F)) = f(F)$. Thus , $f(F) = \mathcal{N} D \propto cl(f(F))$ i.e. , $f(F)$ is $\mathcal{N}$ closed in $\mathcal{V}$. Then , f is $\mathcal{N}$ $D \propto$ closed function .

Theorem: 2.10

Let $f:(\mathcal{U},\mathcal{T}_R(X)) \to (\mathcal{V},\mathcal{T}_{R'}(Y))$ $\mathcal{N}$ D$\propto$ open iff $f(\mathcal{N} D \propto \text{int}(\aleph)) \subseteq \mathcal{N} D \propto \text{int}(f(\aleph))$, $\forall \aleph \subseteq \mathcal{U}$.

Definition: 2.11

c is called $\mathcal{N}$ $D \propto$ homeomorphism if

(i) f is one to one and onto.
(ii) f is $\mathcal{N}$ D$\propto$ continuous.
(iii) f is $\mathcal{N}$ D$\propto$ open.

Theorem: 2.12

Each $\mathcal{N}$ continuous function is $\mathcal{N} \propto$ continuous .

Proof: Let $f:(\mathcal{U},\mathcal{T}_R(X)) \to (\mathcal{V},\mathcal{T}_{R'}(Y))$ *is* $\mathcal{N}$ –continuous, then, $\exists \aleph \in \mathcal{N}$-open in $\mathcal{U}$. $\mathcal{N}$int $\aleph$=$\aleph$. Then $\mathcal{N}$cl($\mathcal{N}$int $\aleph$)= $\mathcal{N}$cl($\aleph$) $\supseteq \aleph$. That is $\aleph \subset \mathcal{N}$cl($\mathcal{N}$int $\aleph$). Therefore, $\mathcal{N}$int $\aleph \subseteq \mathcal{N}$int($\mathcal{N}$cl($\mathcal{N}$int $\aleph$))). That is, $\aleph \subseteq \mathcal{N}$int ($\mathcal{N}$cl ($\mathcal{N}$int $\aleph$))). Thus, $\aleph$ is $\mathcal{N} \propto$-open. Hence, $\mathcal{N} \propto -$continuous.

Theorem: 2.13

Each $\mathcal{N} \propto$-continuous function is $\mathcal{N} \mathcal{D} \propto$-continuous.

Proof: Obvious.

Remark: 2.14

The converse of the above theorem is not true.

3- $\mathcal{N}$ contra continuous function.

Ganster and R. [10] proposed and investigated the concept of N continuous function in 1989. A function : $(\mathcal{U},\mathcal{T}) \to (Y,\mathcal{V})$ If the preimage of every open set is closed, is called $\mathcal{N}$ contra-continuous.

Definition: 3.1

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 3200-3205
https://publishoa.com
ISSN: 1309-3452
Let ($\mathcal{U}$, $\mathcal{T}_R$(X)) and ($\mathcal{V}$, $\mathcal{T}_R$(Y)) be a $\mathcal{N}$ topological space , then f : ($\mathcal{U}$, $\mathcal{T}_R$(X)) → ($\mathcal{V}$, $\mathcal{T}_R$(Y)) is a $\mathcal{N}$ contra D∝ continuous, if f^{-1}(P) is $\mathcal{N}$ D∝ closed in ($\mathcal{U}$, $\mathcal{T}_R$(X)) , ∀$\mathcal{N}$ open set P in ($\mathcal{V}$, $\mathcal{T}_R$(Y)) .

Theorem: 3.2

Each $\mathcal{N}$ contra ∝ continuous is $\mathcal{N}$ contra D ∝ continuous function.

Theorem: 3.3

Each $\mathcal{N}$ contra continuous is $\mathcal{N}$ contra D∝ continuous.

Remark: 3.4

The opposite of the preceding theorem is false.

Example: 3.5

Let $\mathcal{U} = \{p, r, z, w\}$, with $\mathcal{U} / R = \{\{p, r\}, \{z\}, \{w\}\}$, $X = \{p, z\}$, then $\mathcal{T}_R(X) = \{ \mathcal{U}, \emptyset, \{z\}, \{p, r, z\}, \{p, r\}\}$,and $Y = \{p, r\}$.Then $\mathcal{T}_{R'}(Y) = \{ \mathcal{U}, \emptyset, \{p\}, \{p, r, w\}, \{r, w\}$. and $\mathcal{T}_R^c$(X) = $\{\emptyset, \mathcal{U}, \{p, r, w\}, \{w\}, \{z, w\}\}$.Then $\mathcal{N}$ g-closed(X) =$\{ \mathcal{U}, \emptyset, \{w\}, \{p, w\}, \{r, w\}, \{z, w\}, \{p, r, w\}, \{r, z, w\}, \{p, z, w\}\}$, $\mathcal{N}g$-open(X) is compiled of $\mathcal{N}g$-closed(X). $\mathcal{T}_R^{D\propto}\, O(X) = \{\emptyset, \mathcal{U}, \{p\}, \{r\}, \{z\}, \{p, r\}, \{p, z\}, \{r, z\}, \{p, r, z\}, \{r, z, w\}, \{p, z, w\}\}$,

$[\mathcal{T}_R^{D\propto}\, O(\mathrm{X})]^C = \{\emptyset, \mathcal{U}, \{r, z, w\}, \{p, z, w\}, \{p, r, w\}, \{z, w\}, \{r, w\}, \{p, w\}, \{w\}, \{p\}, \{r\}\}$.

$\mathcal{T}_{R'}^c$(Y)={ $\mathcal{U}$,∅,{r,z,w},{z},{p,z}}.

Define f :($\mathcal{U}, \mathcal{T}_R$(X)) → ($\mathcal{U}, \mathcal{T}_{R'}$(Y)) as $f(p) = p$, $f(r) = r$, $f(z) = z$, $f(w) = w$. Then $f^{-1}(\mathcal{U}) = \mathcal{U}$, $f^{-1}(\emptyset)$= $\emptyset$,$f^{-1}(p) = \{p\} \notin \mathcal{T}_{R'}^c(Y)$, $f^{-1}(r) = \{r\} \notin \mathcal{T}_{R'}^c(Y)$. ∴ $\mathcal{N}$ contra D ∝ continuous is not $\mathcal{N}$ contra continuous function.

We summarize the foregoing theories in the following diagram

[$\mathcal{N}$ contra continuous → $\mathcal{N}$ contra ∝ continuous→ $\mathcal{N}$ contra D∝ continuous.]

Now we will review $\mathcal{N}$ Slightly continuous function. The concept of a $\mathcal{N}$ slightly continuous function is introduced, and characterizations and several D ∝ continuous and fundamental features of a $\mathcal{N}$ slightly continuous function [11] are examined and derived.

Definition: 3.6

Let ($\mathcal{U}$, $\mathcal{T}_R(X)$) and ($\mathcal{V}$, $\mathcal{T}_R(Y)$) be a $\mathcal{N}$ topological space , then f : ($\mathcal{U}$, $\mathcal{T}_R(X)$) → ($\mathcal{V}$, $\mathcal{T}_R(Y)$) is a $\mathcal{N}$ slightly D ∝ continuous, at a point $x \in X$ if ∀$\mathcal{N}$clopen subset V in Y containing $f(x)$, ∃ D ∝ $\mathcal{N}$ open subset U in X containing x s.t. $f(U) \subseteq V$.

Example: 3.7

Let $\mathcal{U}$ ={ ɯ,q, m , k} with $\mathcal{U}/R$ ={{ ɯ },{ q, k},{m}}.let X={ ɯ,m, k} ⊆ U. Then $\mathcal{T}_R$(X)={$\mathcal{U}$, ∅, {q, k}, { ɯ, m}}, and $\mathcal{T}_R^{D\propto}\, O(X)$ = {∅,$\mathcal{U}$,{ ɯ }, { q }{ m}, { k}, { ɯ, q }, { ɯ, m}, {q, m}, {q, k}, { ɯ, k}, { m, k}, { ɯ, q, m}, { ɯ, q, k}, {q, m, k}

,{ ɯ, m, k}}. Let $\mathcal{V} = \{a, b, c, d\}$, and with $/ R' = \{\{a\}, \{b, d\}, \{c\}\}$, and $Y = \{a, b, c\}$. Then $\mathcal{T}_{R'}(Y) = \{\mathcal{V}, \emptyset, \{a, c\}, \{b, d\}\}$, and $\mathcal{T}_{R'}^c$(Y) $= \{\emptyset, \mathcal{V}, \{b, d\}, \{a, c\}\}$. $\mathcal{N}$clopen set in Y.

Define f :($\mathcal{U}, \mathcal{T}_R$(X)) → ($\mathcal{V}, \mathcal{T}_{R'}$(Y)) as f(ɯ) = a, $f(q) = b$, $f(m) = c$, $f(k) = d$.

. Then $f^{-1}(a)$ = { ɯ, m} ∈ $\mathcal{T}_R^{D\propto}\, O(X)$,and $f^{-1}(b, d) = \{t, k\} \in \mathcal{T}_R^{D\propto}\, O(X)$.

∴ f is $\mathcal{N}$ slightly D∝ continuous.

Theorem: 3.8

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 3200-3205
https://publishoa.com
ISSN: 1309-3452

Let $(\mathcal{U},\ \mathcal{T}_R(\mathrm{X}))$ and $(\mathcal{V},\ \mathcal{T}_R(\mathrm{Y}))$ be a $\mathcal{N}$ topological space , then $f\ :(\mathcal{U},\ \mathcal{T}_R(\mathrm{X}))\rightarrow(\mathcal{V},\ \mathcal{T}_R(\mathrm{Y}))$ a function . Adjectives are equivalent .

(1) f is $\mathcal{N}$ slightly $D\propto$ continuous.
(2) $\forall\mathcal{N}$clopen $\mathrm{V}\subseteq\mathrm{Y}$, $f^{-1}(V)$ is $\mathcal{N}\ D\propto$ open X.
(3) $\forall\ \mathcal{N}$clopen $\mathrm{V}\subseteq\mathrm{Y}$, $f^{-1}(V)$ is $\mathcal{N}\ D\propto$ closed X.
(4) $\forall\mathcal{N}$clopen $\mathrm{V}\subseteq\mathrm{Y}$, $f^{-1}(V)$ is $\mathcal{N}\ D\propto$ clopen X.

Theorem: 3.9

Any $\mathcal{N}$ contra $D\propto$ continuous is $\mathcal{N}$ slightly $D\propto$ continuous function .

Proof: It easy.

Theorem: 3.10

Each $\mathcal{N}$ slightly $\mathrm{D}\propto$ continuous function is $\mathcal{N}$ $\mathrm{D}\propto$ continuous.

Proof: Let $\mathrm{f}:(\mathcal{U},\ \mathcal{T}_{\mathrm{R}}(\mathrm{X}))\ \rightarrow(\mathcal{V},\mathcal{T}_{\mathrm{R}}(\mathrm{Y}))$ be a $\mathcal{N}$slightly $\mathrm{D}\propto$ continuous function. Let p a $\mathcal{N}$-open in Y . Then $\mathrm{f}^{-1}(\mathrm{p})$ is $\mathcal{N}\mathrm{D}\propto$-open in X. And $\mathcal{N}$ $\mathrm{D}\propto$ closed in X . Hence, f is $\mathcal{N}$ $\mathrm{D}\propto$ continuous.

We will summarize the above theories in the following diagram

[$\mathcal{N}$continuous → $\mathcal{N}$ slightly continuous → $\mathcal{N}$ slightly D∝ continuous → $\mathcal{N}$ D∝ continuous.]

Now initiate the new concept of $\mathcal{N}$Irresolute, $\mathcal{N}$ D∝ continuous, and $\mathcal{N}$ Irresolute D∝ continuous.

Definition: 3.11

Let $(\mathcal{U},\ \mathcal{T}_R(X))$ and $(\mathcal{V},\ \mathcal{T}_R(Y))$ be a $\mathcal{N}$ topological space , then $f\ :(\mathcal{U},\ \mathcal{T}_R(X))\rightarrow(\mathcal{V},\ \mathcal{T}_R(Y))$ is said a $\mathcal{N}$ Irresolute $D\propto$ continuous if $\mathrm{f}^{-1}(\mathrm{O})$ is a $\mathcal{N}$ $\mathrm{D}\propto$ open in $(\mathcal{V},\mathcal{T}_{\mathrm{R}}(\mathrm{Y}))$ $\forall\mathcal{N}$ $\mathrm{D}\propto$ open set O in $(\mathcal{U},\mathcal{T}_{\mathrm{R}}(\mathrm{X}))$.

Theorem: 3.12

Each $\mathcal{N}$ -Irresolute function is $\mathcal{N}$ -continuous.

Theorem: 3.13 Let f: $(\mathcal{U},\mathcal{T}_{\mathrm{R}}(\mathrm{X}))\rightarrow(\mathcal{V},\mathcal{T}_{\mathrm{R}'}(\mathrm{Y}))$ is $\mathcal{N}$ $\mathrm{D}\propto$-Irresolute iff f^{-1} $\mathcal{N}$ $\mathrm{D}\propto$ closed of $(\mathcal{V},\mathcal{T}_{\mathrm{R}'}(\mathrm{Y}))$ is $\mathcal{N}$ $\mathrm{D}\propto$ closed set in $(\mathcal{U},\mathcal{T}_{\mathrm{R}}(\mathrm{X}))$.

Proof: Assume that f is $\mathcal{N}$ $\mathrm{D}\propto$-Irresolute. Let S be any $\mathcal{N}$ $\mathrm{D}\propto$ closed in $(\mathcal{V},\mathcal{T}_{\mathrm{R}'}(\mathrm{Y}))$. So S^{c} $\mathcal{N}$ $\mathrm{D}\propto$-open in $(\mathcal{V},\mathcal{T}_{\mathrm{R}'}(\mathrm{Y}))$. Since , f is $\mathcal{N}$ $\mathrm{D}\propto$-Irresolute , $\mathrm{f}^{-1}(\mathrm{S}^{\mathrm{c}})$ is $\mathcal{N}$ $\mathrm{D}\propto$-open in $(\mathcal{U},\mathcal{T}_{\mathrm{R}}(\mathrm{X}))$. But $\mathrm{f}^{-1}(\mathrm{S}^{\mathrm{c}})=\mathrm{U}\ /\ \mathrm{f}^{-1}(\mathrm{S})$ and so , $\mathrm{f}^{-1}(\mathrm{S})$ is $\mathcal{N}$ $\mathrm{D}\propto$ closed in $(\mathcal{U},\mathcal{T}_{\mathrm{R}}(\mathrm{X}))$. So , $\mathrm{f}^{-1}(\aleph)$ for every $\aleph$– closed $\mathcal{V}$ is $\mathcal{N}$- $\mathrm{D}\propto$ closed $\aleph$ in $\mathcal{U}$. Let C any D∝-open in $(\mathcal{V},\mathcal{T}_{\mathrm{R}'}(\mathrm{Y}))$ .Then C^{c} is $\mathcal{N}$ $\mathrm{D}\propto$ closed $(\mathcal{V},\mathcal{T}_{\mathrm{R}'}(\mathrm{Y}))$ .By assumption , $\mathrm{f}^{-1}(\ \mathrm{C}^{\mathrm{c}})$ is $\mathcal{N}$ $\mathrm{D}\propto$ closed $(\mathcal{U},\mathcal{T}_{\mathrm{R}}(\mathrm{X}))$ .But $\mathrm{f}^{-1}(\mathrm{C}^{\mathrm{c}})$ $\mathcal{N}$ $\mathrm{D}\propto$ closed set in $(\mathcal{U},\mathcal{T}_{\mathrm{R}}(\mathrm{X}))$. So, f $\mathcal{N}$ $\mathrm{D}\propto$-Irresolute .

Theorem: 3.14

Every $\mathcal{N}\mathrm{D}\propto$ Irresolute map is $\mathcal{N}\mathrm{D}\propto$ continuous.

Proof: Let f: $(\mathcal{U},\mathcal{T}_{\mathrm{R}}(\mathrm{X}))\rightarrow(\mathcal{V},\mathcal{T}_{\mathrm{R}'}(\mathrm{Y}))$ a $\mathcal{N}$ $\mathrm{D}\propto$-Irresolute. Let p a $\mathcal{N}$-open set in $\mathcal{V}$. Then p is $\mathcal{N}$ $\mathrm{D}\propto$-open in $\mathcal{U}$. Since f is $\mathcal{N}\mathrm{D}\propto$ Irresolute. $\mathrm{f}^{-1}(\ p)$ is $\mathcal{N}$ $\mathrm{D}\propto$-open in $\mathcal{U}$. So, f is $\mathcal{N}\mathrm{D}\propto$ continuous.

Remark: 3.15

The opposite of the above theory is incorrect .

Example:3.16

Let $\mathcal{U}=\{\mathrm{u},\mathrm{q},\mathrm{p},\mathrm{y}\}$ with $\mathcal{U}$ /R={{u}, {p}, {q, y}}and $\mathrm{X}=\{\mathrm{u},\mathrm{q}\}$.then the $\mathcal{N}$ topolog, $\mathcal{T}_{\mathrm{R}}(\mathrm{X})=\{\mathcal{U},\emptyset,\{\mathrm{u},\mathrm{q},\mathrm{y}\},\{\mathrm{u}\},\{\mathrm{q},\mathrm{y}\}\}$.the $\mathcal{T}_{\mathrm{R}}^{\mathcal{D}\propto}\mathrm{O}(\mathrm{X})=\{\emptyset,\mathcal{U},\{\mathrm{u}\},\{\mathrm{q}\},\{\mathrm{y}\},\{\mathrm{u},\mathrm{q}\},\{\mathrm{q},\mathrm{y}\},\{\mathrm{u},\mathrm{y}\},\{\mathrm{u},\mathrm{q},\mathrm{p}\},\{\mathrm{u},\mathrm{q},\mathrm{y}\},\{\mathrm{u},\mathrm{p},\mathrm{y}\}\}$. And Let $\mathcal{V}=\{\mathrm{k},\mathrm{y},\mathrm{o},\mathrm{t}\}$ with $\mathcal{V}\setminus\mathrm{R}'$={{k, o}, {y}, {t}}

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 3200-3205
https://publishoa.com
ISSN: 1309-3452
and $Y = \{k, y\}$. Then $\mathcal{T}_{R'}(Y) = \{\mathcal{V}, \emptyset, \{\{k, y, o\}, \{y\}, \{k, o\}\}$. Define $f: \mathcal{U} \rightarrow \mathcal{V}$ as $f(u) = k$, $f(q) = y$, $f(p) = t$, $f(y) = t$. $\mathcal{T}_{R'}^{D\propto} O(y) = \{ \mathcal{V}, \emptyset, \{k\}, \{y\}, \{o\}, \{k, y\}, \{k, o\}, \{y, o\}, \{k, y, o\}, \{k, y, t\}, \{y, o, t\}\}$. Then,

$f^{-1}(y) = \{q\} \in \mathcal{T}_{R}^{D\propto} O(X)$ .Then f $\mathcal{N}$ D $\propto$ continuous. But , $f^{-1}(y, o, t) = \{q, p, y\} \notin \mathcal{T}_{R}^{D\propto} O(X)$. Hence, f is $\mathcal{N}$ D $\propto$ continuous but not $\mathcal{N}$ D∝-Irresolute.

References

[1] M. L. Thivagar, C. Richard, On nano continuity, Math. Theory Model. 7 (2013) 32-37.

[2] D. A. Mary, I. Arockiarani, On characterization of nano rgb-closed sets in nano topological spaces, Int. J. Mod. Eng. Res. 5 (1) (2015) 68-76.

[3] D. A. Mary, I. Arockiarani, On b-open sets and b-continuous functions in nano topological spaces, Int, J. Innov. Res. Stud. 3 (11) (2014) 98-116.

[4] P. Anbarasi Rodrigo, K. Rajendra Suba. Contra β^*- Continuous Functions in Topological Spaces, Int, J. of Mathematics Trends and Technology (IJMTT) – Volume 55 Number 8 – March (2018).

[5] G. Arul Gesti, P. Suganya. $\propto_{Ng}$- Irresolute Functions in Nano Topological Spaces, Int, J. of innovative Science, Engineering, Technology, Vol. 8 lssue 1, January (2021).

[6] Lellis Thivagar. M, Saeid Jafari and Sutha Devi. V, On new class of contra continuity in nano topology, Italian Journal of pure and Applied Mathematics-N. 43-2020 (25-36).

[7] Hadi, M.H., Al-Yaseen, M.A.A.-K. , “Study of Hpre -open sets in topological spaces ,” AIP Conference Proceedings 2292 , 2020.

[8] Al-Swidi, L.A., Mustafa, H.H. , “Characterizations of continuity and compactness with respect to weak forms of ω -Open sets ,” European Journal of Scientific Research 57(4), pp. 577-582 , 2011.

[9] Hadi, M.H., AL-Yaseen, M.A.A.-K., Al-Swidi, L.A. , “Forms weakly continuity using weak ω–open sets ” Journal of Interdisciplinary Mathematics ,24(5), (2021), pp. 1141–1144.

[10] GANSTER, M. and REILLY, Locally closed sets and LC-continuous function, Internat. J. Math. Math. Sci., 3 (1989), 417-424.

[11] Raja Mohammad Latif. Slightly Continuous Function in Topological Spaces, Int, J. of pure Mathematics, Volume 6 (2019).